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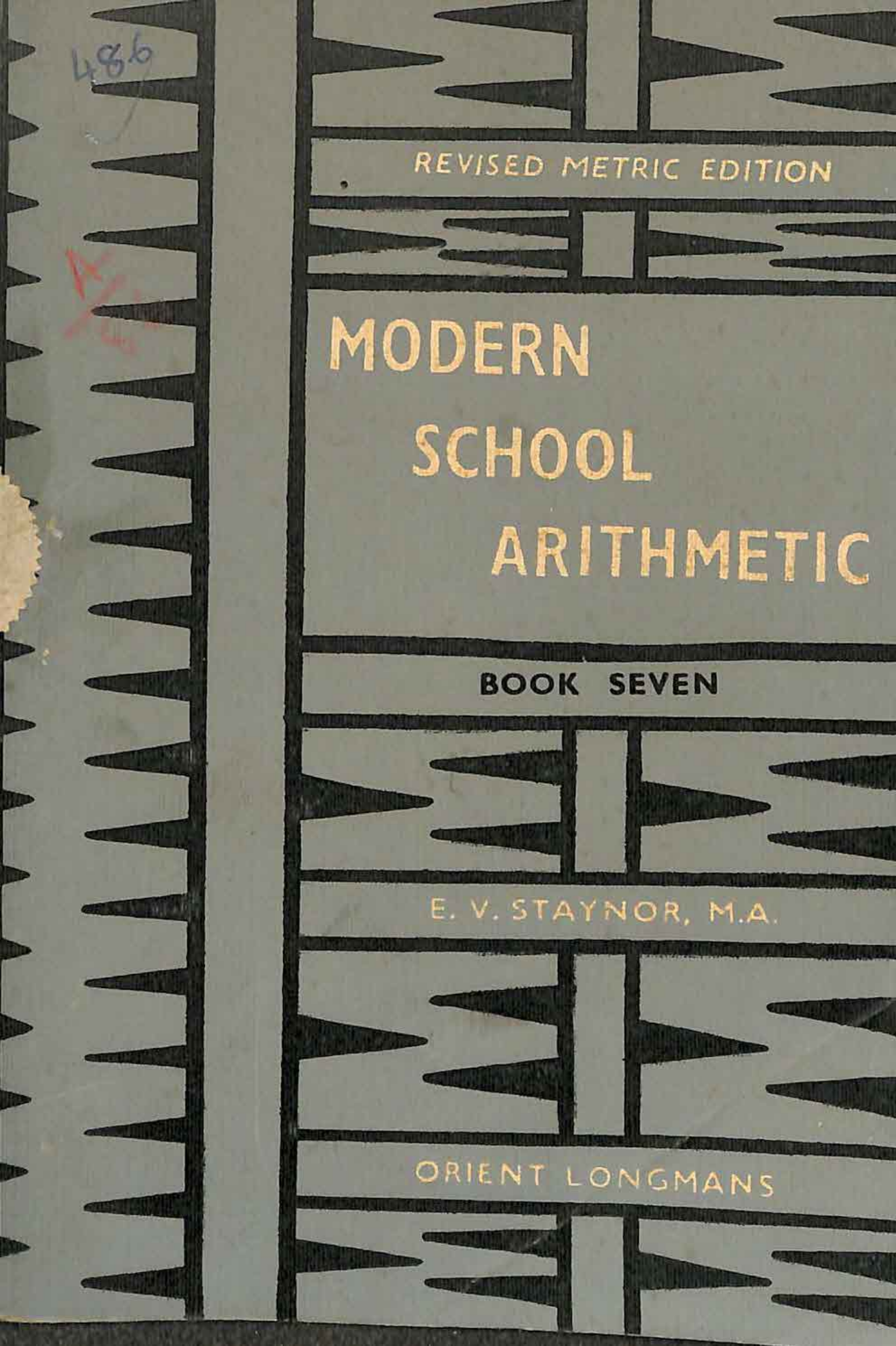

REVISED METRIC EDITION

MODERN SCHOOL ARITHMETIC

BOOK SEVEN

E. V. STAYNOR, M.A.

ORIENT LONGMANS

486 (~~3560~~)

MODERN SCHOOL ARITHMETIC

REVISED METRIC EDITION

BOOK SEVEN

A/66

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PREFACE TO REVISED EDITION 1958

This, together with the other books in the Modern School Arithmetic series, has been revised to cater for the change-over to the new Decimal Money System. The main body of the revised text is continuous with the revised texts of the earlier books.

A discussion and explanation of the teaching plan adopted for teaching Decimal Money has been published in a separate brochure obtainable from the Publishers.

I would like to express my deep gratitude to Mr. T. S. Rajagopalan, B.A., L.T., Assistant, Hindu High School, Triplicane, Madras, for attending to the general setting-out of the text and for many valuable suggestions for clarifying and improving.

E. V. S.

PREFACE TO THE REVISED EDITION 1963

The book has been revised and rearranged so as to conform with the Core Mathematics syllabus of Class IX, leading to the Indian School Certificate Examination in Class XI. The book contains problems both on the Metric System and British weights and measures.

The use of tables of Squares and Square Roots has been introduced. A chapter on Logarithms has also been included with a large number of examples.

A separate section on Trigonometry has been added which should prove useful as an introduction to the Mathematics Syllabus of the School Certificate Examination.

Revision test papers at the end have been rewritten so that they may be used as 80-minute revision tests.

R. Bentinck

MEASURES AND WEIGHTS

METRIC

LENGTH

10 millimetres (mm) = 1 centimetre (cm)
10 centimetres (cm) = 1 decimetre (dm)
10 decimetres (dm) = 1 metre (m)
10 metres (m) = 1 dekametre (dam)
10 dekametres (dam) = 1 hectometre (hm)
10 hectometres (hm) = 1 Kilometre (km)

AREA

100 sq.mm = 1 sq.cm
100 sq.cm = 1 sq.dm
100 sq.dm = 1 sq.m

VOLUME

1000 cu. mm = 1 cu. cm (c.c.)
1000 cu. cm = 1 cu. dm
1000 cu. dm = 1 cu. metre

CAPACITY

10 centilitres = 1 decilitre
10 decilitres = 1 litre
1 litre = 1 cu. dm = 1000 c.c.

WEIGHT

10 milligrams (mg) = 1 centigram (cg)
10 centigrams = 1 decigram (dg)
10 decigrams = 1 gram (gm)
10 grams = 1 dekagram (dag)
10 dekagrams = 1 hectogram (hg)
10 hectograms = 1 Kilogram (kg)
1000 Kilograms = 1 metric tonne

1 cc. of water weighs 1 gram.
1 litre of water weighs 1 Kilogram.

TABLES

MONEY

BRITISH

4 farthings (*f* or *q*) = 1 penny (*d*)
12 pence = 1 shilling (*s*)
20 shillings = 1 pound (£)
1 crown = 5*s*.
1 guinea = 21*s*.

INDIAN

100 naye Paise = 1 Rupee (Re.)

LENGTH

12 inches = 1 foot (ft)
3 feet = 1 yard (yd)
22 yards = 1 chain (ch.)
10 chains = 1 furlong (fur.)
8 furlongs = 1 mile (m.)

1760 yards = 1 mile
220 yards = 1 furlong

AREA

144 $(=12^2)$ sq. inches = 1 sq. foot
9 $(=3^2)$ sq. feet = 1 sq. yard
4840 sq. yards = 1 acre (ac.)
640 acres = 1 sq. mile

VOLUME

1728 $(=12^3)$ cu. in. = 1 cubic ft
27 $(=3^3)$ cu. ft = 1 cubic yd

CAPACITY

2 pints = 1 quart (qt)
4 quarts = 1 gallon (gal.)
8 pints = 1 gallon

TABLES

WEIGHT (Avoirdupois)

16 ounces = 1 pound (lb.)
28 pounds = 1 quarter (qr)
4 quarters = 1 hundredweight (cwt)
20 hundredweights = 1 ton (t)
112 pounds = 1 hundredweight
2240 pounds = 1 ton
14 pounds = 1 stone

1 gallon of water weighs 10 lb.
1 cu. ft of water weighs 1000 oz. or 62·5 lb. approx.

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CHAPTER I

H.C.F. and L.C.M.

EXERCISES 1

1. Find the prime factors of the following numbers and then write down their H.C.F. and L.C.M.:

(*a*) 108; 72 (*b*) 144; 288; 48 (*c*) 63; 28; 35

(*d*) 340; 357; 187 (*e*) 1700; 255; 765.

Find the L.C.M. of 18, 63, 84, 108.

$$\begin{array}{r|rrrr} 2 & 18, & 63, & 84, & 108 \\ \hline 2 & \cancel{9}, & 63, & 42, & 54 \\ \hline 3 & & 63, & 21, & 27 \\ \hline 3 & & 21, & \cancel{7}, & 9 \\ \hline & & 7 & & 3 \end{array}$$

9 omitted as it is a common factor of 63 and 54.

7 omitted as it is a factor of 21

L.C.M. $2^2 \times 3^3 \times 7 = 756$. Ans.

2. Use the above method to find the L.C.M. of the following numbers, leaving your answer in the factor form, if the answer is too large:

(*a*) 15, 25, 27, 45

(*b*) 22, 18, 33, 27

(*c*) 39, 91, 56, 104

(*d*) 40, 35, 85, 119, 136

(*e*) 28, 36, 44, 63, 77

(*f*) 20, 38, 45, 57, 95

(*g*) 69, 105, 115, 161

(*h*) 32, 35, 70, 80, 112

(*i*) 48, 49, 56, 84, 91.

3. What is the least sum of money that can be divided up into equal sums of £ 23 11*s*. or £ 39 5*s*.?

4. Find the least number which is a multiple of 27 or 36.

5. Find the least number which is a multiple of 45 or 63.

6. Find the least number which when divided by 27 or 36 leaves a remainder of 4.

7. Find the least number which when divided by 45 or 63 leaves a remainder of 3.

8. Find the least number which when divided by either 26 or 39 will leave a remainder of 2.

9. Find the least number which when increased by 10, will be exactly divisible by both 14 and 21.

10. Three bells toll at intervals of 15 sec., 27 sec., 36 sec. respectively. If they toll together at 10 A.M. when will they next toll together?

11. Find the greatest number that will divide 2070 and 2755 with remainder 15.

12. Find a number that will divide each of the numbers 1088, 1627, 1480 with remainder 10.

13. Four bells toll at intervals of 5, 6, 12 and 15 seconds beginning together. After how long will they again toll together? How many times would each bell have tolled in this interval?

14. If the H.C.F. of two numbers is 7 and their L.C.M. is 140, find the possible numbers.

15. When a certain number x is divided by 11 the remainder is 1. Prove that if x is divided by 22 the remainder must be 1 or 12.

CHAPTER II

FRACTIONS

Example 1.

Worked Simplifications

$$
\begin{aligned}
& \tfrac{1}{4}(1\tfrac{1}{3}+1\tfrac{1}{2})\div[1\tfrac{2}{5}(\tfrac{1}{8}-\tfrac{1}{12})] \\
= {} & \tfrac{1}{4}(2+\tfrac{1}{3}+\tfrac{1}{2})\div[\tfrac{7}{5}(\tfrac{1}{8}-\tfrac{1}{12})] \\
= {} & \tfrac{1}{4}(2+\tfrac{2+3}{6})\div[\tfrac{7}{5}(\tfrac{3-2}{24})] \\
= {} & \tfrac{1}{4}(2\tfrac{5}{6})\div(\tfrac{7}{5}\times\tfrac{1}{24}) \\
= {} & \frac{1}{\cancel{4}}\times\frac{17}{\cancel{6}}\times\frac{5}{7}\times\frac{\cancel{24}}{1} \\
= {} & \tfrac{85}{7} \\
= {} & 12\tfrac{1}{7}. \text{ Ans.}
\end{aligned}
$$

Example 2.

$$
\begin{aligned}
& 3\tfrac{1}{2}-1\tfrac{1}{2}\times 1\tfrac{3}{4}+2\tfrac{1}{4} \\
= {} & 3\tfrac{1}{2}-\tfrac{3}{2}\times\tfrac{7}{4}+2\tfrac{1}{4} \\
= {} & 3\tfrac{1}{2}-\tfrac{21}{8}+2\tfrac{1}{4} \\
= {} & 3\tfrac{1}{2}-2\tfrac{5}{8}+2\tfrac{1}{4} \\
= {} & 3-2+2+\tfrac{1}{2}-\tfrac{5}{8}+\tfrac{1}{4} \\
= {} & 3+\tfrac{4-5+2}{8} \\
= {} & 3+\tfrac{1}{8} \\
= {} & 3\tfrac{1}{8}. \text{ Ans.}
\end{aligned}
$$

Example 3.

$$
\begin{aligned}
& 5\tfrac{2}{3}\div(2-\tfrac{1}{2}) \text{ of } \tfrac{2}{15}\div 1\tfrac{1}{5} \\
= {} & 5\tfrac{2}{3}\div 1\tfrac{1}{2} \text{ of } \tfrac{2}{15}\div 1\tfrac{1}{5} \\
= {} & \frac{17}{3}\div\frac{\cancel{3}}{\cancel{2}} \text{ of } \frac{\cancel{2}}{\underset{5}{\cancel{15}}}\div\frac{6}{5} \\
= {} & \tfrac{17}{3}\div\tfrac{1}{5}\div\tfrac{6}{5} \\
= {} & \tfrac{17}{3}\times\tfrac{5}{1}\times\tfrac{5}{6} \\
= {} & \tfrac{17\times 25}{18} \\
= {} & \tfrac{425}{18} \\
= {} & 23\tfrac{11}{18}. \text{ Ans.}
\end{aligned}
$$

Example 4.

$$
\begin{aligned}
& \tfrac{7}{30} \text{ of } (\tfrac{1}{2}+\tfrac{7}{15})\div(\tfrac{5}{6}-\tfrac{3}{5}) \\
= {} & \tfrac{7}{30}\times(\tfrac{15+14}{30})\div(\tfrac{25-18}{30}) \\
= {} & \frac{\cancel{7}}{30}\times\frac{29}{\cancel{30}}\times\frac{\cancel{30}}{\cancel{7}} \\
= {} & \tfrac{29}{30}. \text{ Ans.}
\end{aligned}
$$

EXERCISES 2

Simplify

1. $\frac{4}{5}+\frac{3}{5}$ of $[\frac{25}{29}-\frac{4}{13}]$
2. $\frac{4}{5}+\frac{3}{5}\times\frac{25}{39}-\frac{4}{13}$
3. $(\frac{1}{3}-\frac{1}{4})\,(\frac{4}{5}-\frac{5}{7})\div(\frac{1}{3}-\frac{2}{5}+\frac{1}{7})$

4. $(2\frac{1}{2}-1\frac{5}{8})(2\frac{1}{4}+1\frac{7}{8})\times\frac{4}{7}$
5. $1\frac{3}{5}+2\frac{2}{3}+3\frac{7}{10}$
6. $\frac{6}{7}-\frac{2}{5}(\frac{1}{2}+\frac{2}{11})$
7. $\frac{4}{15}$ of $1\frac{2}{3}-\frac{1}{12}$ of $\frac{2}{5}$
8. $\frac{4}{15}$ of $[\frac{1}{7}-\frac{1}{12}]$ of $1\frac{2}{5}$
9. $5\frac{2}{3}\div(2-\frac{1}{2})+(\frac{2}{15}\div1\frac{1}{5})$
10. $(5\frac{1}{5}-3\frac{1}{3})\div\left(1-\dfrac{1\frac{3}{4}}{2\frac{11}{12}}\right)$
11. $\dfrac{2\frac{1}{5}-1\frac{3}{4}}{\frac{1\frac{1}{2}}{5}\div2\frac{1}{2}}$
12. $\dfrac{6\frac{1}{2}-4\frac{3}{4}}{3\frac{1}{2}-2\frac{1}{3}}$
13. $\frac{57}{22}[\frac{3}{5}+\frac{4}{9}$ of $3\frac{3}{5}]-4$
14. $\dfrac{1+\frac{1}{2}-\frac{1}{3}-\frac{1}{4}}{1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}}$
15. $\dfrac{2\frac{1}{6}-\frac{15}{16}}{1\frac{3}{5}+2\frac{1}{3}}$
16. $\dfrac{2\frac{7}{9}-\frac{5}{12}+\frac{43}{36}}{9\frac{1}{3}\times4\frac{5}{7}\text{ of }\frac{4}{33}}$
17. $\dfrac{(6\frac{1}{5}-5\frac{3}{4})\div\frac{3}{70}}{(\frac{1}{4}+\frac{1}{18})\div(\frac{5}{8}+\frac{1}{4})}$
18. $\dfrac{11-(\frac{4}{7}\text{ of }17)}{4-2\frac{4}{5}}-\dfrac{4-2\frac{4}{5}}{11-(\frac{4}{7}\text{ of }17)}$
19. $(2\frac{3}{4}+1\frac{1}{10})\div(2\frac{7}{12}-1\frac{3}{10})$
20. $\dfrac{1\frac{3}{5}+1\frac{7}{8}}{5\frac{19}{21}-3\frac{1}{4}}\div16\frac{17}{25}$.

EXERCISES 3

1. Evaluate:

   (*a*) $\frac{4}{5}$ of 2*s*. 6*d*.
   (*b*) $\frac{7}{12}$ of £ 2 5*s*.
   (*c*) $\frac{9}{16}$ of Rs. 247·20

(*d*) $\frac{5}{17}$ of 34 hrs 35 min. 25 sec.

(*e*) $\frac{8}{25}$ of 455 metres.

2. Express the first quantity as a fraction of the second in the following:

(*a*) 7*s*. 3*d*; 9*s*. 8*d*.

(*b*) 2 tons 2 cwt 2 qr.; 5 tons.

(*c*) Rs 135·60; Rs 180·80.

(*d*) 2904 sq. yds; 1 acre.

(*e*) £6. 11*s*. 3*d*.; £15 15*s*.

(*f*) 2 sq. ft 18 sq. in. 1 sq. yd.

(*g*) 2 ft 9 in.; $\frac{1}{4}$ mile.

(*h*) 252 lb.; $1\frac{1}{2}$ tons.

3. A boy saves half his monthly pocket money for expenses to be met at the end of term. He spends a sixth on a cinema ticket and the remainder, which is Rs 2·50, on confectionery. How much pocket money does he receive a month?

4. The fifth part of a certain number exceeds the sixth part of the number by 4. What is the number?

5. A sum of money is divided among three boys so that the first gets half of it and the second a third of the remainder. What fraction of the first boy's is the third boy's share?

6. A boy, after giving away $\frac{1}{4}$ of his money to a friend and $\frac{7}{8}$ of the remainder to another, has Rs 9 left. How much had he at first?

Method:

He gives $\frac{1}{4}$ of his money. ∴ he has $\frac{3}{4}$ left.
He gives $\frac{7}{8}$. ∴ he has $\frac{1}{8}$ left.
∴ he has $\frac{1}{8}\times\frac{3}{4}=\frac{3}{32}$ left.
But $\frac{3}{32}$ is equivalent to Rs. 9 (given).

∴ The whole is equivalent to $\dfrac{\overset{3}{\cancel{9}}\times 32}{\cancel{3}}$ =Rs 96. Ans.

7. A carpenter cuts off from a plank $\frac{5}{12}$ of its length and then $\frac{3}{7}$ of what remains. If the remaining piece is $2\frac{1}{2}$ metres long, find the original length of the plank.

8. A girl spends $\frac{2}{5}$ of her pocket money, and then $\frac{5}{12}$ of the remainder is given to her brother. If she has £1 1*s*. left, what had she at first?

9. A cistern is full of water. If $\frac{3}{7}$ of it is emptied first, and then $\frac{1}{4}$ of the remainder is drawn out, there are 39 litres left. What is the capacity of the cistern?

10. A man spends $\frac{1}{10}$ of his income on taxes and $\frac{2}{3}$ of the remainder on personal expenses and has Rs 135 left. What is his income?

11. A money prize is divided between A, B, C; A receives $\frac{2}{5}$ of the prize, B receives $\frac{5}{9}$ of the remainder. If C's share is Rs 48, find the value of the prize.

12. A man left $\frac{5}{16}$ of his money to his wife, $\frac{7}{11}$ of the rest to his daughter and the balance of Rs 1600 to charity. What amount did his daughter receive?

13. One quarter of a school playground is unfit for use; two-fifths of the remainder is kept for foot-ball and this leaves 27 acres for new buildings. What is the area of the field?

14. 7/15 of the water in a tank which is full is used on one day, and 5/6 of the remainder on the next. If 80 litres are then left, how much does the tank hold?

15. A radiogram loses $\frac{1}{4}$ of its value in its first year, and in the second year loses $\frac{1}{6}$ of its value at the beginning of that year. At the end of the second year it is worth Rs 1400; what was its value when new?

CHAPTER III

DECIMALS

EXERCISES 4

1. Express the following decimal fractions as vulgar fractions in their lowest terms:

0·34; 0·575: 0·375; 0·875; 0·3125; 0·1115; 0·6125.

2. Express the following vulgar fractions as decimal fractions (correct to the nearest third place):

$\frac{3}{4}$; $\frac{13}{14}$; $\frac{2}{3}$; $\frac{4}{9}$; $\frac{355}{113}$; $\frac{22}{7}$; $\frac{4}{7}$; $\frac{4}{13}$; $\frac{4}{11}$; $\frac{11}{15}$; $\frac{11}{25}$; $\frac{512}{625}$.

Example 1. Express $\frac{2}{5}$ as a percentage.

A percentage fraction or percentage is a fraction with its denominator 100; only, this denominator is written 'per cent' instead of in actual figures. *Per cent*, is short for 'per centum'.

$$\frac{2}{5}=\frac{2\times100}{5\times100}$$

(*The lower 100 is now written as "per cent".*)

$=\frac{2}{5}\times100$ per cent

$=40$ per cent.

Example 2. Express $\frac{12}{23}$ as a percentage fraction or percentage.

$\frac{12}{23}=\frac{12}{23}\times\frac{100}{100}$

$=\frac{1200}{23}$ per cent.

$=52\frac{4}{23}$ per cent.

3. Express the following vulgar fractions as percentages:

$\frac{4}{7}$; $\frac{2}{3}$; $\frac{3}{5}$; $\frac{47}{100}$; $\frac{11}{13}$; $\frac{19}{25}$; $\frac{5}{11}$; $\frac{23}{37}$; $\frac{27}{65}$; $\frac{53}{100}$; $\frac{127}{1000}$; $\frac{121}{178}$.

4. Convert the following percentages to vulgar fractions in their lowest terms:

$66\frac{2}{3}$ per cent; 55 per cent; $12\frac{1}{2}$ per cent; $54\frac{6}{11}$ per cent;
$\frac{3}{7}$ per cent; $\frac{4}{11}$ per cent; $57\frac{1}{7}$ per cent; $61\frac{7}{13}$ per cent;
$36\frac{2}{3}$ per cent; 56 per cent; $58\frac{14}{27}$ per cent; $29\frac{7}{17}$ per cent;
$8\frac{1}{3}$ per cent, $44\frac{4}{9}$ per cent; $52\frac{4}{23}$ per cent; $\frac{1}{10}$ per cent;
72 per cent; $7\frac{1}{5}$ per cent.

5. Arrange each of the following groups in descending order of magnitude:

(*a*) $\frac{2}{7}$; $\frac{3}{8}$: $\frac{1}{3}$; $\frac{4}{11}$; $\frac{7}{19}$.

(*b*) $\frac{3}{8}$; 0·4; 36 per cent.

(*c*) 0·33; $\frac{1}{3}$; $\frac{8}{25}$; $\frac{13}{40}$.

(*d*) $\frac{22}{7}$; 3·1416; $\frac{355}{113}$.

(*e*) $\frac{9}{20}$; $\frac{25}{51}$; $\frac{8}{19}$; $\frac{17}{39}$.

(*f*) 0·756; $\frac{187}{250}$; 75 per cent.

(*g*) $\frac{1}{3\cdot142}$; 0·318.

(*h*) $\frac{4}{3\cdot7}$; $\frac{9}{25}$.

(*i*) $7\frac{4}{11}$; $7\frac{1}{2}$; $7\frac{5}{13}$; $7\frac{2}{5}$; 7·39.

(*j*) 32 per cent; $\frac{1}{3}$; $\frac{11}{34}$; $\frac{12}{37}$; 0·33.

(*k*) $\frac{21}{25}$; 0·87; $\frac{38}{45}$; $\frac{59}{70}$; $\frac{16}{19}$.

WORKED EXAMPLES

Example 3.

Express 13 shillings $10\frac{1}{2}$ pence as a decimal of £1.

$10\frac{1}{2}d = 10{\cdot}5d$

$12\overline{)10{\cdot}5d}(\cdot875s.$

$20\overline{)13{\cdot}875s.}$

$0{\cdot}69375£$

£0·69375 Ans.

Example 4.

What decimal fraction of 4 tons 15 cwt 2 qrs. is 2 tons 13 cwt 1 qr.?

tons	cwt	qrs.	tons	cwt	qrs.
4	15	2	2	13	1
=	95	2	=	53	1
=		382	=		213

The required fraction $= \frac{213}{382}$

$= 0{\cdot}56$ to 2 decimal places. Ans.

$$\begin{array}{r} 0{\cdot}557 \\ 382\overline{)213{\cdot}0} \\ 191{\cdot}0 \\ \hline 22{\cdot}00 \\ 19{\cdot}10 \\ \hline 2900 \end{array}$$

Example 5.

What decimal fraction is £ 2 3*s*. 9*d*. of £ 2 12*s*. 1*d*. ?

$$\text{Fraction} = \frac{£2\ 3s.\ 9d.}{£2\ 12s.\ 1d.}$$

$$= \frac{£2\ 3\frac{3}{4}s.}{£2\ 12\frac{1}{12}s.}$$

$$= \frac{£2\frac{3}{16}}{2\frac{29}{48}}$$

$$= \frac{\overset{7}{\cancel{35}}}{\cancel{16}}\times\frac{\overset{3}{\cancel{48}}}{\underset{25}{\cancel{125}}}=\frac{21}{25}=0{\cdot}84.\ \text{Ans.}$$

EXERCISES 4 (*Contd*)

6. What decimal fraction is
   (*a*) Rs 234·60 of Rs 361?
   (*b*) Rs 180·30 of Rs 240·4?
   (*c*) £2 12*s*. 6*d*. of £7?
   (*d*) 17 dollars 5 cents of 31 dollars 25 cents (2 dec. places)?
   (*e*) 3 gal. 1qt 1 pt of 13 gal. 2 qt (2 dec. places)?

7. Express £1 14*s*. 10*d*. as a decimal of £4 (3 dec. places).

8. Express £ 4 15*s*. 5*d*. as a decimal of £5 (3 dec. places).

9. Express 6 hrs 15 min. 22 sec. as decimal of 24 hours correct to 3 dec. places.

10. Simplify (*a*) $\dfrac{36{\cdot}8 \times 0{\cdot}005}{1{\cdot}472}$

    (*b*) $\dfrac{5{\cdot}5 \times 0{\cdot}175}{0{\cdot}275 \times 12{\cdot}5}$.

11. Find in inches the difference between $0{\cdot}27\frac{3}{4}$yds and 0·834 ft.

12. Simplify $(4{\cdot}3)^2+(4{\cdot}1)^2-8(2{\cdot}1)^2$.

13. Find p and q if $p+q=0{\cdot}2$ and $q=\frac{1}{2}p$.

14. If 1 gallon is approximately equal to 4·546 litres, express 50 gallons in litres to the nearest litre.

15. Express 11 quarts in litres correct to the nearest centilitre (1 gal. = 4·546 litres).

16. If 0·9144 metres is approximately equal to 1 yd, express 7920 feet in metres to the nearest metre.

17. Express £2 12*s*. 9*d*. as a fraction of £4 11*s*.

18. Express 1200 yds as a fraction of 12 furlongs.

19. Evaluate:
    (*a*) $\frac{9}{14}$ of 199 tons 17 cwt.
    (*b*) $\frac{13}{25}$ of £444 13*s*. 9*d*.
    (*c*) $\frac{9}{16}$ of Rs 247·20.
    (*d*) $\frac{5}{17}$ of 34 hrs 35 min. 25 sec.
    (*e*) $\frac{4}{7}$ of 199 sq. yds 1 sq. ft.

20. Find the value of:
    $\frac{3}{40}$ of Rs 71·60 $-\frac{7}{50}$ of Rs 12·50 $+\frac{2}{25}$ of Rs 11·75.

21. From $\frac{13}{15}$ of 266 tons 16 cwt 1 qr take away $\frac{11}{20}$ of 355 tons 15 cwt 1 qr.

22. Which is greater, $\frac{10}{11}$ of Rs. 19.36 or $\frac{9}{14}$ of Rs 25·20? By how much ?

23. Work out the following scheme and test the equivalence of the last line.

	Rs		Rs
	$\frac{4}{17}$ of 154·70	=	....
	$\frac{3}{13}$ of 154·70	=	....
Add	.. of 154·70	=	....

24. Express as £ *s.* *d.*

(*a*) £31·7625

(multiply the decimal by 20) 15·250*s.*

(,, ,, ,, by 12) 3·00*d.*

£31 15*s.* 3*d.* Ans.

(*b*) £53·8125 (*c*) £32·3186 (*d*) £49·003

(*e*) £72·0605 (*f*) £64·753.

Express as tons cwt qr.

(*g*) 5·735 tons (*h*) 7·354 tons.

Express as hours min. sec.

(*i*) 7·112 hours (*j*) 73·256 hours.

25. What vulgar fraction is:

(*a*) £ 106 19*s.* of £139 10*s.*?

(*b*) Rs 135·60 of Rs 180·80?

(*c*) 995 lb. 10 oz. of 1261 lb. 2 oz.?

(*d*) 26 hrs 19 min. 10 sec. of 54 hrs 44 min. 40 sec.?

CHAPTER IV

PRACTICE

Example 1.

Find the total weight of 83 loads each 5 cwt 1 qr. 14 lb.

		cwt	qr.	lb.
	Weight at 1 cwt per load	83	0	0
	Weight at 5 cwt per load	415	0	0
1 qr $=\frac{1}{4}$cwt.	Weight at 1 qr. per load	20	3	0
14 lb. $=\frac{1}{2}$ qr.	Weight at 14 lb. per load	10	1	14
	Weight of 5 cwt 1 qr 14 lb. per load	446	0	14

446 cwt 14 lb.

=22 tons 6 cwt 14 lb. *Ans.*

Example 2.

		£	*s.*	*d.*
Multiply £3 19*s*. 9*d*. by 124				
124 times £4		496	0	0
3*d*=1/80 of £1 ∴ 124 × 3*d*.	Subtract	1	11	0
£ 3 19*s*. 9*d*. × 124 is		494	9	0

£494 9 shillings. *Ans.*

Example 3.

A train travels 37 miles an hour. How far, to the nearest one-hundredth of a mile, does it go in 1 hr 38 minutes 25 sec.?

	in 1 hour, distance	=	37 miles.
30 min. $=\frac{1}{2}$ of 1 hr.	in 30 min.	,,	18·5
5 min. $=\frac{1}{6}$ of 30 min.	in 5 min.	,,	3·0833
3 min. $=\frac{1}{10}$ of 30 min.	in 3 min.	,,	1·85
20 sec. $=\frac{1}{9}$ of 3 min.	in 20 sec.	,,	·2055
5 sec. $=\frac{1}{4}$ of 20 sec.	in 5 sec.	,,	·0513

in 1 hr 38 min. 25 sec. distance=60·6901 miles

60·69 miles. *Ans.*

Example 4.

Multiply Rs 3·74 by $5\frac{5}{16}$

			Rs.
$\frac{5}{16}=\frac{1}{4}+\frac{1}{16}$	5 times	Rs. 3·74	=18·70
	$\frac{1}{4}$ of	,,	= 0·935
	$\frac{1}{16}=\frac{1}{4}$ of $\frac{1}{4}$	,,	= 0·2337$\cancel{5}$*
	$6\frac{5}{16}$ times	,,	=19·8687$\cancel{5}$
			Rs 19·87. *Ans.*

*NOTE.—In examples such as this the answer is required to the nearest nP and it is only necessary to work as far as the fourth place of decimals in Rs.

EXERCISES 5

1. Find by practice the total weight of the following:
   (*a*) 166 loads each 5 cwt. 1 qr 14 lb.
   (*b*) 68 loads each 1 ton 12 cwt 2 qrs.
   (*c*) 840 loads each 1 lb. 11 oz.
   (*d*) 87 loads each 2 tons 13 cwt 1 qr.
   (*e*) 135 loads each 2 tons 17 cwt 2 qrs.

2. Find by practice the cost of 132 tons 10 cwt of coal @ £4 3*s*. 6*d*. per ton.

3. Find the cost of 74 shirts at £1 17*s*. 9¼*d*.

4. Find the cost of 37 long playing records at £2 6*s*. 2*d*. each.

5. A car travels 27 miles an hour; find, to the nearest tenth of a mile, the distance it travels in 47 min. 15 sec.

6. A car travels 38 km an hour; find, to the nearest tenth of a km, the distance it travels in 3 hours 27 min. 25 sec.

7. Multiply.
   (*a*) 3 hours 40 min. 25 sec. by 67.
   (*b*) 2 hours 48 min. 20 sec. by 81.
   (*c*) 5 hours 35 min. 30 sec. by 74.
   (*d*) 4 hours 26 min. 40 sec. by 57.

(*e*) 4 yds 2 ft 7 in. by 77.
(*f*) 3 yds 1 ft 9 in. by 120.
(*g*) 8 yds 1 ft 11 in. by 135.

8. Find the cost of 315 articles at £2 18*s*. 9*d*. each.

9. Use subtraction methods to find

(*a*) the cost of 87 metres of ribbon at $5\frac{1}{2}$ *d*. per metre.
(*b*) the weight of 126 boxes each weighing 5 lb. 7 oz.
(*c*) the cost of 195 yards at 11*d*. per yard.
(*d*) the cost of 692 metres at 1*s*. $10\frac{1}{2}$*d*. per metre.

10. Find the cost of 105 nibs at Rs 3 per gross.
$(105=72+24+9=\frac{1}{2}$ gross $+\frac{1}{6}$ gross $+\frac{1}{16}$ gross.)

11. Find the total weight of 87 truck-loads at 2 tons 13 cwt 1 qr per truck-load.

12. Multiply (by practice)
17*s*. 9*d*. by $3\frac{5}{8}$; $4\frac{11}{16}$; 3.8; $4\frac{11}{28}$; $2\frac{5}{12}$; $5\frac{4}{9}$.
Give your answers to the nearest penny.

EXERCISES 6

Find by practice the cost of

1. 4 tons 8 cwt at £2 10*s*. per ton.
2. 11 lb 6 oz. at £ 3 8*s*. per lb.
3. 3 tons 5 cwt i qr at £ 4 6*s*. 8*d*. per ton.
4. 5 gallons 3 qt 1 pt at Rs 31·30 per gallon.
5. 7 gallons 2 qt at Rs 25·25 per gallon.

6. The market price of a certain kind of metal is Rs 2050 a ton. Find the cost of each of the following lots (to the nearest nP.):

(*a*) 2 tons 15 cwt 1 qr.
(*b*) 1 ton 13 cwt 3 qrs.
(*c*) 3 tons 17 cwt 2 qrs.
(*d*) 352 tons 7 cwt 1 qr.
(*e*) 213 tons 3 cwt 3 qrs.

7. What is the cost of 50 yds 2 ft 9 in. of fencing at 3*s*. 9*d*. a yard (to the nearest penny)?

8. Find the cost of each of the following quantities of coal at £2 7*s*. 9*d*. a ton. (to the nearest penny):

(*a*) 5 tons 7 cwt 3 qrs.
(*b*) 12 tons 17 cwt 1 qr.
(*c*) 50 tons 16 cwt 2 qrs.
(*d*) 157 tons 11 cwt 1 qr.
(*e*) 216 tons 13 cwt 3 qrs.

9. Find the weight of 27 cub. ft 1008 cub. in. of wood, if 1 cub. ft weighs 25 lb. 3 oz.

10. Find the weight of 8 sq. in. of metal sheeting, if 1 sq. yd weighs 23 lb. 7 oz.

11. Find the cost of 6 st. $5\frac{3}{4}$ lb. of mineral at Rs 78·50 a cwt.

12. Find the cost of 5 st. $6\frac{3}{8}$ lb. of mineral at Rs 85·70 a cwt.

CHAPTER V

SQUARE ROOT

To find the square root of 1849, we can first put the problem in this way: The area of a square is 1849 sq. units; how long is its side?

Let ABCD be the given square of area 1849 sq. units. (Fig. 1*a*.)

We know that AB is greater than 40 units; for 40^2 is 1600. We know that AB is not as great as 50 units; for 50^2 is 2500. We now mark off AX=40 and draw the square AXYZ. The area of AXYZ is 1600 sq. units, and the area of the remainder, the shaded part in the figure, must be 1849—1600, that is 249 sq. units. Since XB=ZD, we can lay this shaded part out in the form of a rectangle (Fig. 1*b*.)

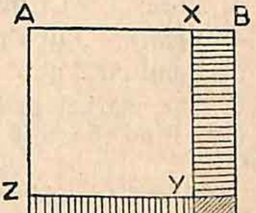

Fig. 1*a*.

This consists of two equal rectangles and a square.

Now XY and YZ are each equal to 40 units.

The long side of the rectangle is therefore greater than twice 40. Taking this side to be twice 40, that is, 80, the width XB$=\frac{249}{80}=3$(about). Now the side of the small square is equal to XB. So we take XZ (Fig. 1*b*.) to be 83 units. The area of the rectangle is now 83×3 sq.units, which gives us 249 sq. units exactly.

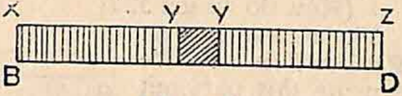

Fig. 1*b*.

Our work can be displayed compactly as follows: We first mark off the digits in pairs (starting from units and working to the left). (We shall see why presently.)

The nearest square to 18 is 16, so we take the square root of 16, that is 4, for the 'tens' figure of our answer and place it in position. (Remember that this is really the 40 we used above.) Now double the 4 (really 40) and write it down as a divisor. (Why do we double the 4?) 8 goes into 24, 3 times. We place 3 as the units figure besides the divisor 8 and also in the answer. Multiplying 83 by 3, we get 249. We see that $\sqrt{1849}$ is 43 exactly.

$$\begin{array}{r|l} & \text{4 3 Ans.} \\ \hline 4 & 1849 \\ & 16 \\ \hline 83 & 249 \\ & 249 \\ \hline \end{array}$$

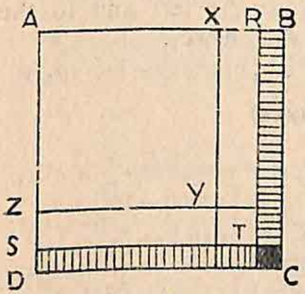

Fig. 2*a*.

Find $\sqrt{2743}$.

Consider a square of area 2743 sq. units (Fig. 2*a*).

Take AX=50. Why? The area of Fig. 2*b* is 2743−2500, that is 243.

Now XZ is greater than twice 50. Why?

If it were twice 50, we would get XB$=\frac{248}{100}=2$ (about).

So we take XB=2 and XZ=102. (Fig. 2*b*.) This time we have not got the area exactly as 2×102, which equals 204, is 39 short of 243. Evidently XB is somewhat greater than 2.

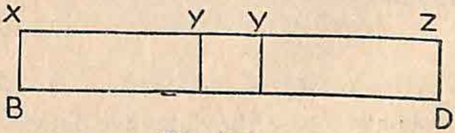

Fig. 2*b*.

Going back to fig. (2*a*), however, we take AR=52.
(How do we get 52?)

The area of the part RBCDST is 39. We can lay this part out as in fig. 2*c*. Now RS is greater than twice 52.

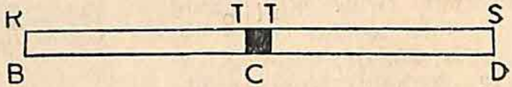

Fig. 2*c*.

If it were twice 52, that is 104, RB would be $\frac{39}{104}=0{\cdot}3$ (about).

So RB=0·3 and RS=104·3. This, however, would give us an area of 104·3×0·3, which is equivalent to 31·29.

As this is not exactly 39, we have not found $\sqrt{2743}$ exactly. We can say, however, that the figures 52·3 are correct as far as they go. To find the figures after 3, we can go on repeating the process already adopted. We shall learn later, however, that the decimal answer neither terminates nor recurs. We can, of course, find the answer correct to any number of decimal places as may be required.

Here is the process displayed in compact form and the square root found to three decimal places. Study the example and notice, firstly, the zeros are filled in as required after the decimal point; secondly, that the digits are paired, starting from the units figure and working outwards to the left and to the right. Each pair yields one digit towards the answer.

		5 2· 3 7 3
	5	2743·000000
		25
double 5	102	243
		204
double 52	1043	39·00
		31·29
double 523	10467	7·7100
		7·3269
double 5237	104743	·383100
		·314229
		·068871
		52·373.... Ans.

52·373 gives the first five figures of $\sqrt{2743}$. Notice that the last line (·06...) shows us how much out we would be in the area of the square if we took the side to be exactly 52·373.

EXERCISES 7

Find the sq. root of:
(1) 529 (2) 729 (3) 841 (4) 1024
(5) 1521 (6) 2209 (7) 2809 (8) 5041
(9) 6084 (10) 7225 (11) 9801 (12) 64009
(13) 97969 (14) 4284900 (15) 5006001.

Find the Square root correct to 2 places of decimals:
(16) 8345 (17) 679 (18) 7532·64
(19) 896·9 (20) 7·009 (21) 29·16.

Find the Square root of:
(22) 5·29
(23) 7·29
(24) 10·24
(25) 0·0784
(26) 0·023104
(27) 0·0019184.

Table of Squares

By direct multiplication $218 \times 218 = 47524$.

$\therefore 218^2 = 47524$.

This result can be obtained correct to four figures from four-figure tables (Use Cambridge 4-figure mathematical tables):

Main Columns

	0	1	2	3	4	5	6	7	8	9
21	4410	4452	4494	4537	4580	4623	4666	4709	4752	4796

In your table of squares look for the number 21 in the LEFT-HAND Column; the figures in the "21" row together with the headings at the top of the page are given above.

In the 21 row under the 8 in the main columns you find the figures 4752. This means that the first 4 significent figure is 218^2 are 4752.

Since $200^2 = 40{,}000$; and $300^2 = 90{,}000$ the square of 218 lies between 40,000 and 90,000.

$\therefore 218^2 = 47520$ correct to 4 figures.

The figures obtained when working out the square of 21·8 are exactly the same as those obtained in working out the square of 218, the only difference being the position of the decimal point.

Now $20^2=400$ and $30^2=900$.

∴ the square of 21·8 lies between 400 and 900.

∴ $21{\cdot}8^2=475{\cdot}2$ from the tables and is correct to 4 figures.

Similarly $2{\cdot}18^2 \;=4{\cdot}752$ to 4 figures.

$\;{\cdot}218 \;=0{\cdot}04752$ to 4 significant figures.

EXERCISES (Oral) 8A

Use four-figure tables to find the squares of:

(*a*) 22
(*b*) 35
(*c*) 47
(*d*) 51
(*e*) 62
(*f*) 217
(*g*) 243
(*h*) 238
(*i*) 79·1
(*j*) 7·91
(*k*) ·791
(*l*) 3·17
(*m*) 0·06
(*n*) 0·062
(*o*) 0·0756
(*p*) 0·0101
(*q*) 0·0204
(*r*) 0·0709
(*s*) 0·987
(*t*) 897.

Use of the mean-difference column.

Example. Use tables to find the value of $2{\cdot}187^2$.

An extract from a table of squares showing the 21 row is given on page 17; at the end of this row in the tables the mean differences for that row are added.

MEAN DIFFERENCES

	1	2	3	4	5	6	7	8	9
21	4	9	13	17	21	26	30	34	39

From the main columns we have

$$(2{\cdot}18)^2=4{\cdot}752$$
$$(2{\cdot}19)^2=4{\cdot}796;$$

∴ the value of $(2{\cdot}187)^2$ lies between 4·752 and 4·796.

Under the heading 7 in the mean-difference column we find 30, which means that 30 (here ·030) must be added to 4·752 to obtain the first 4 significant figures in $(2{\cdot}187)^2$.

∴ $2{\cdot}187^2=4{\cdot}782$.

If $2{\cdot}187^2$ is obtained by multiplication, it is 4·782969 so that in the result given by the tables the 4th figure is not completely reliable. The error is not generally more than a unit in the fourth figure. This fact may be shown by writing the result in the form $(2{\cdot}187)^2 = 4{\cdot}78(2)$.

EXERCISES (Oral) 8B

Using your 4-figure tables, find to 4 sig. figures the squares of the following:

(1) 2·87
(2) 2·395
(3) 4·861
(4) 9·324
(5) 8·051
(6) 7·006
(7) 8·013
(8) 4·208
(9) 5·903
(10) 0·712
(11) 0·0134
(12) 0·0101
(13) 0·2151
(14) 0·3019
(15) 9·001
(16) 4·875
(17) 54·925
(18) 72·876
(19) 61·929
(20) 0·30412.

Square-root tables

If a number is multiplied by 10, the digits in its square root are altered, but if it is multiplied by 100, the digits remain the same. Now

$$\begin{aligned} &\sqrt{758} &&= 27{\cdot}53 \\ \therefore\ &\sqrt{7{\cdot}58} &&= 2{\cdot}753 \\ \therefore\ &\sqrt{0{\cdot}0758} &&= 0{\cdot}2753. \\ \text{But}\quad &\sqrt{7580} &&= 87{\cdot}06 \\ &\sqrt{75{\cdot}8} &&= 8{\cdot}706 \\ &\sqrt{0{\cdot}758} &&= 0{\cdot}8706. \end{aligned}$$

For this reason square-root tables are arranged on pairs of pages. On one page the reading corresponding to 758 is 2753 and on the other page it is 8706; decimal points are usually omitted. It is necessary therefore to find by inspection or by the square root rule the FIRST significant figure of the square root and the position of the decimal point.

Use tables to find square root to 4 figures:

(*i*) $\sqrt{497}$ (*ii*) $\sqrt{49{\cdot}7}$

$$\begin{array}{r|l} & 2\ x. \\ \hline 2 & 4,97 \end{array}$$

$\sqrt{497} = 2x.$

Look on the page where the sq. root of 497 starts with 2.

$\therefore \sqrt{497} = 22{\cdot}2(9)$

$$\begin{array}{|l} 7{\cdot}xx \\ \hline 49{\cdot}7 \end{array}$$

$\sqrt{49{\cdot}7} = 7{\cdot}xx$

Look on the page when the sq. root of 497 starts with 7.

$\therefore \sqrt{49{\cdot}7} = 7{\cdot}050$

The mean-difference column is used in the same way for square roots as for squares.

Examples. Use tables to find

1. $\sqrt{5217}$ 2. $\sqrt{0{\cdot}7118}$

(1) $\sqrt{5217} = 7x{\cdot}xx$

$\therefore \sqrt{5217} = 72{\cdot}23$

$$\begin{array}{ll} \sqrt{5210} = & 72{\cdot}18 \text{ (approx.)} \\ \text{Diff. for } 7 & \phantom{72{\cdot}1}5 \\ \hline \sqrt{5217} & 72{\cdot}23 \end{array}$$

72·23 *Ans.*

(2) $\sqrt{0{\cdot}7118} = 0{\cdot}8\ xxx$

$= 0{\cdot}8437$

$$\begin{array}{ll} \sqrt{0{\cdot}7110} = & 0{\cdot}8432 \text{ (approx.)} \\ \text{Diff for } 8 & \phantom{0{\cdot}843}5 \\ \hline \sqrt{0{\cdot}7118} & 0{\cdot}8437 \text{ (approx.)} \end{array}$$

0·8437 *Ans.*

EXERCISES (Oral) 8C

Find to 4 significant figures (using 4-figure tables) the square root of:—

(1) 3·1
(2) 2·9
(3) 4·7
(4) 26
(5) 358
(6) ·358
(7) 75
(8) 750
(9) 7310
(10) 0·0731
(11) 69·8
(12) 0·823
(13) 0·0865
(14) 0·00865
(15) 0·00789
(16) 0·0008176
(17) 0·1016
(18) 2·008
(19) 0·1089
(20) 0·09157.

486

Pythagoras' Theorem

The area of the square on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares on the other two sides.

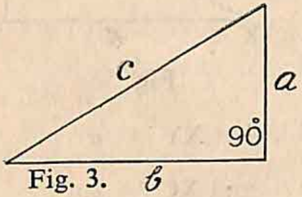

Fig. 3.

$c^2 = a^2 + b^2.$

Example 3.

The lengths of the sides of a rectangle are 4·7 cm and 3·2 cm. Find the length of the diagonal correct to four figures.

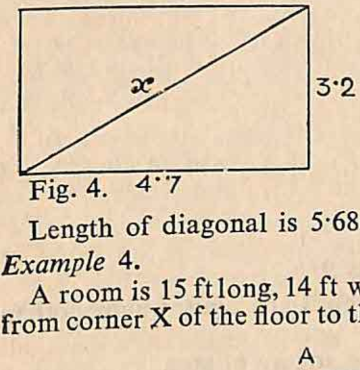

Fig. 4.

Let the diagonal be x cm long. Then by Pythagoras' Theorem

$$x^2 = (3{\cdot}2)^2 + (4{\cdot}7)^2$$
$$= 10{\cdot}24 + 22{\cdot}09 \text{ (table of squares)}$$
$$= 32{\cdot}33;$$
$$\therefore\ x = \sqrt{32{\cdot}33}$$
$$= 5{\cdot}686 \text{ (table of square root)}.$$

Length of diagonal is 5·686 cm. *Ans.*

Example 4.

A room is 15 ft long, 14 ft wide, and 9 ft high. Find the distance from corner X of the floor to the opposite corner Y of the ceiling.

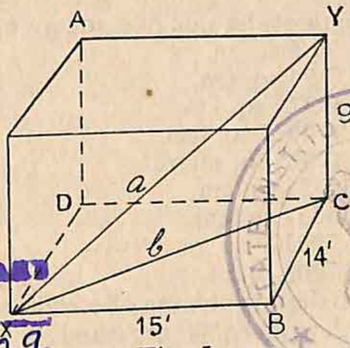

Fig. 5*a*

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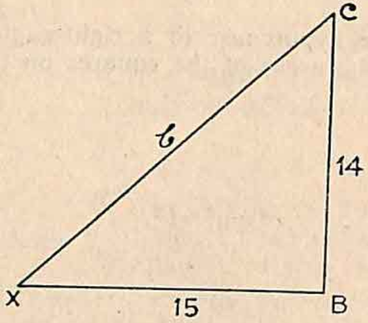

Fig. 5*b*.

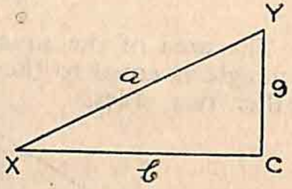

Fig. 5*c*

Let XY $= a$

and XC $= b$.

From right-angled triangle XBC

$$\begin{aligned} b^2 &= 14^2+15^2 \\ &= 196+225 \\ &= 421. \end{aligned}$$

From right-angled triangle XCY

$$\begin{aligned} a^2 &= b^2+9^2 \\ &= 421+81 \\ &= 502 \\ \text{or } a &= \sqrt{502} \\ &= 22{\cdot}41. \end{aligned}$$

22·41 ft. *Ans.*

EXERCISES 9

Give your answers correct to 3 significant figures unless otherwise stated.

(1) Find the length of the side of a square of area

- (*a*) 79·6 sq. in.
- (*b*) 7·96 sq. cm.
- (*c*) 0·325 sq. km.
- (*d*) 0·0625 sq. feet.
- (*e*) 0·625 sq. metre.
- (*f*) 6·25 sq. cm.
- (*g*) 8537 sq. cm.
- (*h*) 796300 sq. m.
- (*i*) 7968 sq. in.

2. The length of the hypotenuse of a right-angled triangle is *c*, and the lengths of the other sides are *a* and *b*. The unit of measurement is cm:

(*a*) If $a = 5$, $b = 12$, find c.
(*b*) If $a = 40$, $c = 41$, find b.
(*c*) If $a = 5·3$, $b = 4·9$, find c.
(*d*) If $a = 6·14$, $b = 4·73$, find c.
(*e*) If $b = 17·12$, $c = 21·23$, find a.

3. Find the diagonal of a cube edge 5 cm.

4. What is the longest stick that can be placed in a box 4 feet by 5 feet by 6 feet?

5. A room is 18 feet long 15 feet wide 10 feet high. What is the length of a diagonal?

6. A square field is 10 acres in area. Find (*i*) the length of its diagonal (*ii*) the cost of fencing it with wire netting at Rs 8·50 a yard.

7. A rectangle has one side five times as long as the other. The area is 100 sq. ft. Find the length of its sides to the nearest inch.

8. A ladder 30 ft long is placed against a wall so that its top is 21 feet from the ground. How far is the foot of the ladder from the wall?

9. A chord of length 15 cm when placed inside a certain circle has its mid-point 10 cm from the centre. What is the radius of the circle?

10. A pole 50 ft high above the ground is to be supported by guy wires joined half way up its height. If the guy wires are to be 35 ft long, how far from the foot of the pole must they be fixed to the ground?

11. Find the length of the diagonals of the rectangular solid whose length, width and thickness are 15, 12, and 8 cm respectively.

12. What is the length of the longest stick that will fit into a rectangular box whose dimensions are 20 cm, 15 cm, and 12 cm?

13. Prove that the triangle whose sides are m^2+n^2, m^2-n^2, and $2mn$ is right-angled.

14. Find the height of an equilateral triangle if the length of each side is 5·6 cm.

15. The length of a chord of a circle of radius 8·2 cm is 14·5 cm. Find the distance of the chord from the centre of the circle.

16. The length of a chord of a circle of radius 6·15 cm is 9·36 cm. Find the distance of the chord from the centre of the circle.

17. The base of a pyramid is a rectangle, 7 cm by 5 cm, and the length of each slant edge of the pyramid is 8 cm. Find the height of the pyramid.

18. A pyramid stands on a square base of side 4 cm. If its height is 9 cm., find the length of the slant edge.

19. The perimeter of an isosceles right-angled triangle is 30 inches. What is the length of the hypotenuse?

20. A man walks 2 km due south, then 5 km due east, then 4 km due south, then 1 km due west. How far is he from his starting point as the crow flies?

21. Evaluate (using tables):

(*a*) $\sqrt{2\frac{14}{25}}$ (*b*) $\sqrt{2\frac{1}{6}}$ (*c*) $\sqrt{3\frac{17}{36}}$

(*d*) $\sqrt{3\frac{3}{5}}$ (*e*) $\sqrt{1\frac{17}{65}}$ (*f*) $\sqrt{11\frac{14}{25}}$

(*g*) $\sqrt{11\frac{3}{7}}$ (*h*) $\sqrt{\frac{1}{7 \cdot 5}}$ (*i*) $\sqrt{3-\frac{3}{4}}$

(*j*) $\sqrt{1+\frac{1}{4}+\frac{5}{16}}$ (*k*) $\sqrt{2(1-\frac{1}{50})}$ (*l*) $\sqrt{\frac{3}{5} \text{ of } \frac{27}{245}}$.

22. Evaluate (to 2 decimal places):

(*a*) $\sqrt{(27)^2+(25)^2}$ (*b*) $\sqrt{(13 \cdot 5)^2-(9 \cdot 6)^2}$

(*c*) $\sqrt{(3 \cdot 7)^2+(4 \cdot 3)^2}$ (*d*) $\sqrt{(1\frac{1}{2})^2+(1\frac{2}{3})^2}$

(*e*) $\sqrt{(7\frac{1}{5})^2-(2\frac{3}{5})^2}$ (*f*) $\sqrt{(36 \cdot 6)^2+(7 \cdot 5)^2}$.

CHAPTER VI

AREA

Rectangle: Circle: Cylinder

To find the area of the four walls of a rectangular room. Imagine the four walls laid out in one plane, thus:

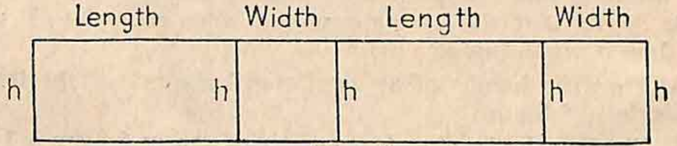

Fig. 6

Notice that they form one big rectangle, one side of which is twice (length+breadth) and the other side is the height.

Thus, the required area is—

$$2(l+b)\times h,$$

and we have the formula $\mathbf{A=2h(l+b)}$.

Example 1.

Find the cost of papering the walls of a room 25 ft long, 23 ft broad, 12 ft high, with paper which is sold in pieces 12 yds long and 21 in. wide and which costs Rs 2·50 a piece.

$$\begin{aligned}\text{Area of walls } &= \text{twice } (l+b)\times h \text{ sq. ft}\\ &= 2\,(25+23)\times 12 \text{ sq. ft}\\ &= 96\times 12 \text{ sq. ft.} \qquad \text{(i)}\end{aligned}$$

The area of each piece of wall paper

$$=12\times 3\times \tfrac{21}{12} \text{ sq. ft.} \qquad \text{(ii)}$$

If we divide (ii) into (i), we get the number of pieces required.

$$\text{The number of pieces } = \frac{96\times 12\times 12}{12\times 3\times 21} = 18\tfrac{2}{7}.$$

We must, of course, buy 19 whole pieces.

The cost, at Rs 2·50 a piece, will be Rs $19\times\frac{5}{2}$

that is Rs $\frac{95}{2}$,

that is Rs 47·50. Ans.

In working the sums that follow, remember

(1) that it is often better to delay multiplying out, or dividing out till the end of the sum, in case later cancellation may simplify calculation.

(2) that not only the use of the 'distributive' law is useful in cutting down work, but its use is expected of you.

(3) not to evaluate a square root, if it is to be squared later on.

Example 2.

Find the area of a square the diagonal of which is 100 units.

If the side is x, we have, by the theorem of Pythagoras

$$2x^2=100^2$$

$$\text{or } x^2= 50.100$$

(Now don't take the square root of 50 to find x.)

Now, x^2 is the area.

So 5000 sq. units. Ans.

Example 3.

In the accompanying figure, ABCD is a square and the arcs are quarter circles, with their centres at A, B, C, and D respectively. AB=14″. Find the area of the shaded portion.

The area of the square is $14^2=196$ sq. in.
Area of the four quarter-circles = a whole circle $=\pi 7^2=22\times 7=154$ sq. in.

The shaded area $=196-154=42$ sq. in.

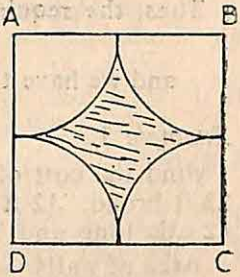

Fig. 7.

EXERCISES 10

1. Find the area of the following rectangles, giving the answer in the units specified after each example:

(*a*) 73 in. by 8·2 in.	(sq. ft.)
(*b*) 7 ft 2 in. by 2 ft 7 in.	(sq. ft, sq. in.)
(*c*) 3 ft 9 in. by 2 ft 8 in.	(sq. ft.)
(*d*) 2 yds 1 ft by 3 yds 2 ft.	(sq. ft.)
(*e*) 13 yds 2 ft by 14 yds 1 ft.	(sq. yds.)
(*f*) 7 miles 440 yds by 8 miles 220 yds.	(sq. miles)
(*g*) 55 metres by 40·4 metres	(Ares)
(*h*) 37·5 cm by 42·8 cm.	(sq. metres)
(*i*) 82·7 cm by 55 cm.	(sq. metres)
(*j*) 3·4 km by 4·3 km.	(Ares)
(*k*) 5·7 km by 2·1 km.	(sq. cm.)
(*l*) 17·4 m. by 8·34 m.	(sq. cm.)

2. The following figures give the area and one side of a rectangle. Find the other side.

	Area	Side
(*a*)	689 sq. yds 4 sq. ft.	73 ft.
(*b*)	9 sq. ft 24 sq. in.	2 ft 9 in.
(*c*)	3 acres 3855 sq. yds.	73·5 yds.
(*d*)	13 acres 1089 sq. yds.	(Square field)
(*e*)	17 sq. ft. 78¼ sq. in.	(Square)
(*f*)	5·9607 Ares.	53·7 metres
(*g*)	0·8722 Ares.	9·8 metres
(*h*)	63·3616 Ares.	(Square)

3. Find the area of the walls of the following rectangular rooms (making no deduction for doors, windows, etc.):

	Length	Breadth	Height
(*a*)	17 ft.	15 ft.	12 ft.
(*b*)	$18\frac{1}{2}$ ft.	17 ft.	12 ft.
(*c*)	$21\frac{3}{4}$ ft.	$18\frac{1}{2}$ ft.	$12\frac{1}{2}$ ft.
(*d*)	25 ft.	$20\frac{1}{2}$ ft.	$11\frac{3}{4}$ ft.

4. Find the cost of papering the walls of a room 18 ft long, 17 ft broad, 12 ft high with wall paper which costs Rs 2·50 for a piece 12 yds long and 21 in. wide.

5. What is the cost of papering the walls of each of the rooms, the dimensions of which are given below, with paper costing Rs 12·40 a piece of 12 metres by 50 cm ?

(*a*) 8 metres × 6 metres × 4 metres.
(*b*) 8·5 metres × 6·5 metres × 4·5 metres.
(*c*) 9·25 metres × 6·5 metres × 4·25 metres.

6. Find the cost of painting the walls of a room, $17\frac{1}{2}$ ft long, 14 ft wide, 13 ft high, at the rate of $2\frac{1}{2}$*d.* a sq. yd, deducting 180 sq. ft for doors and windows.

7. Fill in the missing dimensions in the following :—

All the data refer to rectangular rooms.

	Area of Walls	Length	Breadth	Height
(*a*)	792 sq. ft.	21 ft.	15 ft.	
(*b*)	1196 sq. ft.		$21\frac{1}{2}$ ft.	$11\frac{1}{2}$ ft.
(*c*)	1638 sq. ft.	$34\frac{1}{4}$ ft.		13 ft.
(*d*)	$931\frac{1}{2}$ sq. ft.	Twice as long as it is broad.		$11\frac{1}{2}$ ft.
(*e*)	1092 sq. ft.	Length 5 ft. more than breadth.		12 ft.

8. The cost of painting the walls of a room $23\frac{1}{2}$ ft. long, 15 ft wide, at Rs 1·80 per sq. yd, is Rs 215·60. How high is the room ?

9. The cost of painting the walls of a room 30 ft long and 13 ft. high at Rs 1·35 per sq. yd is Rs 165·75. How wide is the room ?

10. Find the price of an Axminster carpet 12 ft by $10\frac{1}{2}$ ft, selling at Rs 15·75 a sq. yd.

11. Find the cost of completely covering the floor of a room 25 dm × $17\frac{1}{2}$ dm with coir matting at Rs 2·70 a sq. metre.

12. It costs Rs 85·25 to fully mat a room 22′ by $15\frac{1}{2}$′. At what rate per sq. yd is the matting being sold ?

13. It costs 2994 francs 75 cents to fully carpet a room 8·25 m by 5·5 m. What is the price per sq. metre of the carpet ?

14. What is the cost of fencing in a square field $\frac{5}{8}$ acre in area at 3*s*. 6*d*. a yard ?

15. What is the cost of fencing in a square field of area 1225 sq. yd at Rs 3·75 a yard ?

16. If it costs Rs 660 to fence in a square field at Rs 2·50 a yard, find the area of the field, giving your answer as a decimal of an acre.

CHAPTER VII

VOLUME

Volume of Box: Cubical Contents of Material Required.

EXERCISES 11

1. The following are particulars for rectangular solids. Fill in the blanks.

	Length	Breadth	Thickness	Volume
(*a*)	7ft	3 ft 6 in.	2 ft 3 in.	ft^3
(*b*)	$3\frac{1}{8}$ ft	$4\frac{1}{2}$ ft	$2\frac{5}{8}$ ft	ft^3
(*c*)	7 in.	in.	6 in.	399 in^3
(*d*)	0·18 metres	6·3 cm	3·5 c.m.	cc.
(*e*)	7·4 yds	4·5 yds	yd	119·88 yd^3
(*f*)	3 ft 6 in.	4 ft 9 in.	yd	$199\frac{1}{2}$ ft^3
(*g*)	7 m	cm	83 cm	3·3117 m^3
(*h*)	4 yds	3 yds 2 ft	in.	44 ft.^{3}
(*i*)	m	7 cm	8 cm	6384 c.c.
(*j*)	8·3 cm	6·3 m	57 cm	m^3
(*k*)	$13\frac{1}{2}$ ft	$5\frac{1}{2}$ ft	in.	$55\frac{11}{16}$ ft^3
(*l*)	$2\frac{1}{2}$ yds	$2\frac{1}{2}$ ft	in.	$17\frac{3}{16}$ ft^3.

2. How many cubic feet of plaster will be needed to plaster one face of a wall 25 ft long and $13\frac{1}{4}$ ft high, if the plaster is to be laid on $\frac{3}{4}''$ thick ?

3. A rectangular compound 52 ft by 27 ft is to be gravelled to a depth of $3\frac{1}{2}$ in. Find what it will cost at Rs 2·50 per 100 cubic feet.

4. The four sides of a rectangular stone cistern of external dimensions, 4 ft by 3 ft by $5\frac{1}{2}$ ft high, are to be coated outside with $\frac{1}{2}''$ cement plaster. What will it cost at Rs 0·75 a cubic ft ?

5. A rectangular lawn of dimensions 128 ft by 35 ft is flooded with water to a depth of $1\frac{1}{4}$ in. What is the weight of the water on the lawn ? (1 cub. ft. of water weighs 62·5 lb.) (Answer in tons to the nearest $\frac{1}{10}$ ton.)

6. Find the cost of a rectangular piece of timber 12 ft. long, 9 in. wide and $\frac{3}{4}$ in. thick at 2*s*. 3*d*. a cub. ft.

7. Find the cost of 24 planks of wood each 10 ft long, 15 in. wide and $\frac{1}{2}$ in. thick at Rs 2·50 a cub. ft.

8. A wall is to be made 25 ft long, 18 ft high and $\frac{1}{2}$ ft deep with bricks $9''\times6''\times3''$ in size. What will the bricks cost at Rs 3 a hundred ?

9. If a cubic foot of metal is to be made into sheets 6 ft square and $\frac{1}{18}$ in. thick, how many sheets will be obtained ?

10. How many people can be accommodated in a hall, 100 ft long, 35 ft broad and 15 ft high, if each person is allowed 140 cub. ft of air ?

11. How many planks 11′ long, 10″ wide, $\frac{3}{4}''$ thick can be purchased for Rs 178·75 if the wood is being sold at Rs 3·25 a cubic foot ?

12. A solid rectangular block of wood 2 ft long and of square cross-section of side 15 in. weighs 90 lb. What will it weigh after it is planned down $\frac{1}{2}''$ in all its dimensions ?

13. The following are particulars for rectangular solids of square cross-sections. Find the side in each case:

(*a*) Vol. = 79·2 cc height = 5 cm.
(*b*) Vol. = 835·6 ft^3 height = 2·7 ft.
(*c*) Vol. = 1000 ft^3 height = 9·4 ft.
(*d*) Vol. = 856 cc height = 12·6 cm.

14. How much longer must a rectangular solid of height 5·5 in., width 3·7 in. and length 8 in., be for its volume to be 182 cub. inches ?

Example 1.

Find the weight of a lidless box of external dimensions, 15″ by 12″ by 8″ made out of wood $\frac{1}{2}$″ thick weighing 30 lb. to the cubic foot.

The ext. volume $15'' \times 12'' \times 8'' = 1440$ cub. in.
The int. volume $14'' \times 11'' \times 7\frac{1}{2}'' = 1155$ cub. in.
$\therefore$ the volume of wood (subtract) $= 285$ cub. in.
$= \frac{285}{1728}$ cub. ft.
$\therefore$ the weight $= \frac{285}{1728} \times 30$ lb.
$= 4{\cdot}95$ lb.
(approx.)

15. A lidless box of external dimensions, 15 in. by 12 in. by 9 in., made of material 1″ thick, weighs 25 lb. What will it weigh if a flat lid made from the same material is added ?

16. The following are particulars for rectangular lidless wooden boxes. Find the weight of each box.

	External Dimensions. l. b. depth	Thickness of wood.	Wt. of wood.
(*a*)	$5' \times 4' \times 3'$	1″	35 lb. per cub. ft.
(*b*)	$5' \times 3' \times 3'$	$\frac{3}{4}$″	35 lb. ,, ,, ,,
(*c*)	$17'' \times 11'' \times 9''$	$\frac{1}{2}$″	30 lb. ,, ,, ,,
(*d*)	$15'' \times 11'' \times 10''$	$\frac{1}{4}$″	40 lb. ,, ,, ,,
(*e*)	$25'' \times 15'' \times 11''$	$\frac{1}{2}$″	32 lb. ,, ,, ,,
(*f*)	25 cm × 17 cm × 12 cm.	1 cm.	·55 lb. ,, c.c.
(*g*)	30 cm × 25 cm × 10 cm.	1 cm.	·6 gm ,, c.c.

17. Water is contained in a rectangular cistern of cross-section (internal) 6·5 sq. ft. A cube of iron of side $\frac{1}{2}$ ft is dropped into the cistern and is completely submerged. By how many inches will the water level rise ?

18. Water is contained in a rectangular cistern of cross-section (internal) 10 sq. ft. A cube of wood of side $\frac{1}{2}$ ft is put in and floats in the water with 4″ below the surface. By how many inches will the water level rise ?

19. Water is contained to a depth of 9″ in a rectangular cistern of cross-section 6 sq. ft. This water is transferred to another rectangular cistern of cross-section 4 sq. ft. By how much will the water level in the second cistern rise ?

20. Water flows into a rectangular cistern of cross-section 11 sq. ft through a pipe with circular cross-section of diameter 2″, at the rate of 4 feet a second. At what rate per second will the water-level in the cistern rise ?

EXERCISES 12

(1 cubic ft = 6·24 gallons.)

1. Water flows into a tank with square base of side 3·5 feet at the rate of $1\frac{1}{2}$ gallons a minute. How much does the water-level rise in an hour ?

2. How many gallons of water will a tank with internal dimensions 6′×4′×3′ hold ?

3. How many gallons will a cubical tank of side (internal) 7 feet hold ?

4. A rectangular tank with base 3′×2′ is to be constructed to contain 100 gallons. How high should it be ?

5. A tank with a square base is to be made 4 ft high and of 120 gallons capacity. Find the length of the side of the base.

6. A cubical tank is to be built to contain 1000 gallons. What will be the length of its side ? (Take the cube root of 6·24 to be 1·841.)

Example 2.

How many gallons of water will a rectangular tank of internal dimensions 7′×3′×2′ contain ?

(One cubic foot = 6·24 gallons.)

The cubical contents = 7×3×2 cubic ft = 42 cubic ft.
The capacity = 42×6·24 gallons = 262·08 gallons.

Example 3.

A rectangular tank with base 2′×2′ is to be constructed to contain 90 gallons of water. How high should it be ?

The capacity $=90$ gallons.

The cubical capacity (or volume) $=\dfrac{90}{6{\cdot}24}$ cubic feet.

The height $\dfrac{90}{6{\cdot}24\times4}$ feet (divided by the cross-section $2'\times2'$)

$=3{\cdot}6$ feet.

Example 4.

The water-level in a tank, base $2{\cdot}5'\times3'$ and height $4'$, is $2{\cdot}5'$. Water flows in at the rate of 2 gallons a minute. When will the tank start overflowing ?

The volume to be filled $=2{\cdot}5\times3\times(4-2{\cdot}5)$ cubic ft.

The capacity to be filled $=2{\cdot}5\times3\times1{\cdot}5\times6{\cdot}24$ gallons.

But 2 gallons are filled in 1 minute;

$\therefore$ the time taken $=\dfrac{2{\cdot}5\times3\times1{\cdot}5\times6{\cdot}24}{2}$ minutes i.e. in $35{\cdot}1$ min. Ans.

7. Into a tank of cross section 12 sq. ft. 25 gallons of water are poured. By how much will the water-level rise ?
(Take 1 cubic ft. $=6\frac{1}{4}$ gallons.)

8. From a tank of cross-section 15 sq. ft. 20 gallons of water are taken oft. By how much will the water-level fall ?
(1 cubic ft. $=6\frac{1}{4}$ gallons.)

9. Water is poured into an empty tank of dimensions $6'\times5'\times3'$ at the rate of 3 gallons a minute. How long will the tank take to fill ? (1 c. ft. $=6\frac{1}{4}$ gallons.)

10. The water-level in a tank $5'\times4'\times2\frac{1}{2}'$ is only half-way up. Water flows in at the rate of $2\frac{1}{2}$ gallons a minute. When will the tank start overflowing ? (1 c. ft. $=6\frac{1}{4}$ gallons.)

11. In an empty tank $4'\times4'\times2'$ is an outlet pipe half way up from which water can flow at an average rate of 2 gallons a minute. Water enters by a pipe on top of the tank at the rate of 5 gallons a minute. How long will the tank take to fill ? (1 c. ft. $=6\frac{1}{4}$ gallons.)

CHAPTER VIII

CIRCLE

The circumference of a circle of radius r units is $2\pi r$. The area of a circle of radius r units is πr^2 units of area. The area of an annulus is $\pi(R^2-r^2)$ sq. units.

$\pi=\frac{22}{7}$ or 3·14.

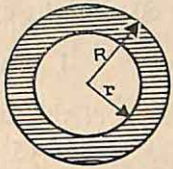

Fig. 8*a*

Example 1.

Find the circumference of a circle of radius 5 inches.

$$\begin{aligned}\text{Circumference} &= 2\pi r\\ &= 2.\tfrac{22}{7}.5\\ &= \tfrac{220}{7}\\ &= \underline{31{\cdot}43 \text{ in.}}\ \text{Ans.}\end{aligned}$$

Example 2.

Find the radius of a circle with a circumference of 100 cm. Let the radius be r cm, then the circumference is $2\pi r$ cm.

$$\begin{aligned}\therefore\ 2\pi r&=100\\ \text{or } r&=\frac{100}{2\pi}=\frac{50}{\pi}\\ &=\frac{50}{3{\cdot}14}\\ &=\underline{15{\cdot}9 \text{ cm.}}\ \text{Ans.}\end{aligned}$$

Example 3.

The diameter of a cycle wheel is 28 inches. Find the number of revolutions made by the wheel per minute when the cycle is travelling at 15 miles per hour.

Circumference of the wheel is $2\pi r$.

$$\begin{aligned} \therefore \text{ Circumference } &= 2\pi.14 \text{ in.} \\ &= 28\pi \text{ in.} \\ &= \frac{28\pi}{12} \text{ ft.} \end{aligned}$$

In 60 min. the cycle travels $15\times1760\times3$ ft.

1 min. the cycle travels $\dfrac{15\times1750\times3}{60}$ ft.

$\therefore$ the number of revolutions made in 1 minute is

$$\frac{15\times1760\times3}{60}\div\frac{28\pi}{12}$$

$$=\frac{15\times1760\times3}{60}\times\frac{12}{28\pi}$$

$$\begin{array}{l} \qquad\quad \cancel{8}2 \qquad 6 \\ =\dfrac{15\times1\cancel{7}\cancel{6}\cancel{0}\times\cancel{3}\times1\cancel{2}\times\cancel{7}}{\cancel{6}\cancel{0}\times\cancel{2}\cancel{8}\times\cancel{2}\cancel{2}}=\underline{\text{180 revolutions.}}\text{ Ans.} \\ \qquad\quad \cancel{2} \quad \cancel{4} \end{array}$$

Example 4.

Find the area of a circle of diameter 9 cm.

The diameter is 9 cm; $\therefore$ radius $=4{\cdot}5$ cm;

$$\begin{aligned} \therefore \text{ area of circle } &= \pi r^2 \\ &= \pi\times\tfrac{9}{2}\times\tfrac{9}{2} \\ &= 3{\cdot}14\times\tfrac{81}{4} \\ &= \frac{3{\cdot}14\times81}{4} \\ &= 63{\cdot}585 \text{ sq. cm} \\ &= \underline{63{\cdot}6 \text{ sq. cm appr.}}\text{ Ans.} \end{aligned}$$

Example 5.

Find the diameter of a circle of area 36 sq. cm.

Area of circle$=\pi r^2$.

$$\therefore \pi r^2 = 36$$

$$\text{or}\quad r^2 = \frac{36}{\pi} = \frac{36}{3{\cdot}14}$$

$$= 11{\cdot}5$$

$$\text{or}\quad r = \sqrt{11{\cdot}5}$$

$$= 3{\cdot}391;$$

$$\therefore \text{diameter} = 2r = 3{\cdot}391 \times 2 = \underline{6{\cdot}782}\text{ cm. Ans.}$$

Example 6.

Find the area of the ring between two concentric circles of radius 4 inches and 5 inches.

Let the larger radius be R inches, and the smaller radius r inches. The area of the ring is $\pi(R^2-r^2)$.

So $R = 5$ inches and $r = 4$ inches.

$$\begin{aligned}\therefore \text{area} &= \pi(R^2-r^2)\\ &= \pi(5^2-4^2)\\ &= \pi(5+4)(5-4)\\ &= \pi\times9\times1\\ &= 9\pi\\ &= 9\times3{\cdot}14\\ &= \underline{28{\cdot}26\text{ sq. inches.}}\text{ Ans.}\end{aligned}$$

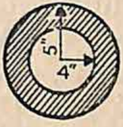

Fig. 8*b*

EXERCISES 13

(In the following examples π is to be taken to be $\frac{22}{7}$. All answers which do not work out exactly are required to the nearest $\frac{1}{10}$ unit.)

1. A cycle is ridden round a track of radius 56 yards. If the diameters of the wheels are 28 inches, find how many turns a wheel makes during one complete lap.

2. A cycle is ridden round a track of diameter 104·3 yard If the diameter of the wheels is $26\frac{8}{40}$ inches, find how many turns a wheel makes during one complete lap.

3. A cycle is ridden round a track of radius 105 feet. A wheel makes 96 revolutions during one complete circuit of the track. What is the diameter of the wheel in inches?

4. What is the cost of fencing a circular plot of ground of radius 70 feet at 7*s.* 6*d.* a yard?

5. What is the cost of fencing a circular plot of radius 14·7 yards at Rs 1·25 a foot?

6. What is the area of a circle of circumference 44 in.?

7. What is the area of a circle of circumference 68·2 ft?

8. What is the cost of fencing a circular plot of ground of area 3850 sq. yards at 3*s.* 8*d.* a yd?

9. What is the cost of fencing a circular plot of ground of area 2464 sq. metres at 18 fr. 35 cent a metre?

10. What is the area of the circle the circumference of which is equal to the perimeter of a square of side 10·5 cm?

11. A wire that has been bent into the shape of a rectangle 6·5″ by 5·4″ is straightened out and then rebent into the form of a square. Which has the larger area, the square or the rectangle? By how much?

12. Answer question (11) for a rectangle 7·5 cm. by 13·2 cm.

13. A piece of wire which has been bent into the form of a circle of radius 3·5 cm is unbent and then bent into the form of a square. In which case does the wire enclose the bigger area? By how much do the areas differ?

14. A piece of wire that has been bent into the form of a semi-circle, including the bounding diameter, is straightened and then bent into the form of a square. The diameter of the circle is 12 cm long. Which area is larger, the semi-circle or the square? By how much?

15. A piece of wire that has been bent into the form of a square of perimeter 20 cm is straightened out and rebent into a circle. What is the area of this circle and by how much does it differ from the area of the square?

16. A circle of diameter 3 cm is drawn on a sheet of paper 9 cm square. If the circle is shaded, what is the area of the unshaded portion? Answer to 3 figures.

17. Draw a circle of radius 4 in. on squared paper (1 inch graph paper). Count the number of $\frac{1}{10}$ inch squares enclosed. Check your answer by calculation.

18. A circular track is prepared for a 400 metre race. Find the radius of the track. Find the length of the side of a square enclosing the same area as the track.

19. The area between the circumferences of two concentric circles of radius 4 cm and x cm is 22 sq. cm. Find the value of x to the nearest integer.

20. A rectangular sheet of metal is 6 cm by 4 cm. Four quadrants of a circle each of radius 2 cm are cut away at the corners. Find the area of what remains.

21. A circular plot of ground is 45 metres in diameter and is surrounded by a cemented path 6 metres in width. Find the cost of cementing the path at Rs 4·75 per sq. metre.

22. A circular plate of metal has a radius of 10 cm and weighs 250 gm. Find the weight of a plate of identical material and thickness, which has a radius 12 cm.

23. Find the radius of a circle the area of which is equal to the sum of the areas of two circles of radius 3 cm and 4 cm.

24. Draw a circle of radius 4 cm with centre O. Draw two radii OX, OY so that angle XOY is 120°. Find the area of the part of the circle between OX and OY. [This area is called a sector of the circle of angle 120°.]

25. Find the area of a sector of a circle of radius b cm, if the angle of the sector is (*a*) 90° (*b*) 150° (*c*) 108° (*d*) 240°.

Circular Cylinder

Area of surface of a circular cylinder.

A rectangular sheet of paper (14″ by 8″) can be rolled into the shape of a circular cylinder with open ends.

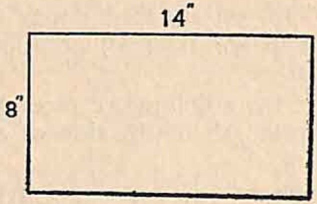

Fig. 9*a*

Fig. 9*b*

Therefore the area of the curved surface of a cylinder
=perimeter of base × height of cylinder
=circumference of base × height of cylinder.

If the radius of a cylinder is r units and if the height of the cylinder is h units, the perimeter of the base is $2\pi r$ units.

$\therefore$ area of curved surface of cylinder
$=2\pi rh$ units of area.

If the TOTAL surface area of a solid cylinder is to be determined, it is necessary to calculate the area of each end. The area of each end is the area of a circle of radius r units and is therefore πr^2 units of area.

$$\therefore \text{ Total area of surface} = (2\pi rh+2\pi r^2) \text{ units of area}$$
$$= 2\pi r(h+r) \text{ units of area.}$$

Example 7.

Find the total area of the surface of a solid cylinder that has a base-radius of 4 cm, and height of 5 cm.

$$\begin{aligned}\text{Total area of surface} &= 2\pi r(h+r) \text{ sq. units}\\ &= 2\pi 4(5+4)\\ &= 2\pi\times4\times9\\ &=72\times\pi\\ &=\frac{72\times22}{7}\\ &=\frac{1584}{7}\\ &= \underline{\underline{226{\cdot}3 \text{ sq. cm.}}} \text{ Ans.}\end{aligned}$$

Volume of circular cylinder=area of cross section×height.

If the radius of the base is r units, the area of its cross section is πr^2 units of area and if the height is h units;

the Volume of cylinder = $\pi r^2 \times h$ units of volume.

Example 8.

Find the volume of water in litres in a cylindrical tank of 4 metres diameter and 1·2 metres depth.

Since the radius is 2 metres and the depth of water is 1·2 metres,

$$\begin{aligned}\text{volume of water} &= \pi r^2 h \text{ cu. m}\\ &= \pi.2^2.1{\cdot}2\\ &= \pi.4{\cdot}8\\ &= 3{\cdot}14 \times 4{\cdot}8\\ &= 15{\cdot}072 \text{ cu. metres. (p. 42)}\\ &= 15072 \text{ cu. dm}\end{aligned}$$

$\therefore$ Volume in litres = 15072. Ans.

Example 9.

A cylindrical tank 5 ft in diameter contains 150 gallons of water. Find the depth of the water. (1 cu. ft = $6\frac{1}{4}$ gallons.)

$$\begin{aligned}\text{150 gallons occupy } & 150 \div 6\tfrac{1}{4} \text{ cu. ft}\\ &= 150 \times \frac{4}{25} = 24 \text{ cu. ft.}\end{aligned}$$

Since Vol. = $\pi r^2 h$,

$$\begin{aligned}\therefore \text{depth of water} &= \text{volume of water} \div \text{area of base}\\ &= 24 \div \pi\frac{25}{4}\\ &= \frac{24 \times 4}{25\pi} = \frac{96}{25\pi} = \frac{96}{25} \times \frac{1}{3{\cdot}14}\\ &= 1{\cdot}223 \text{ ft.}\end{aligned}$$

EXERCISES 14

1. A rectangular sheet of paper 22 inches wide and 10 inches high is rolled into the form of an open hollow cylinder 10 inches high. What is the greatest possible diameter of the cylinder? (Take $\pi = \frac{22}{7}$.) What is the area of the curved surface of the

cylinder? What is the area of the smallest piece of paper which will cover one end of it?

2. A hollow cardboard cylinder is 7 inches in diameter, and the height of the cylinder is 12 inches. A cut is made straight down the surface so that the cylinder can be opened out to form a rectangle. Find the breadth and height and area of the rectangle.

3. Find the area of the curved surface of the following cylinders ($\pi = \frac{22}{7}$):

(*a*) Radius 7 inches, height 4 inches.

(*b*) Radius 4·2 inches, height 8 inches.

(*c*) Radius 2·8 cm, height 6 cm.

(*d*) Radius 5 cm, height 1 dm. ($\pi = 3{\cdot}14$.)

(*e*) Diameter 1 yard, thickness 1 inch. ($\pi = 3{\cdot}14$.)

4. Find the total areas of the surfaces of the solid cylinders. ($\pi = \frac{22}{7}$ or 3·14.)

(*a*) Radius 2 cm, height 5 cm.

(*b*) Radius 5 cm, height 10 cm.

(*c*) Radius 3 feet, height 4 feet.

5. Find the radius of a cylinder of height 5 cm and area of curved surface 110 cm.

6. A garden roller is 2 feet 6 inches in diameter and 3 feet wide. What area does it cover (*a*) in 1 revolution (*b*) in 50 revolutions (*c*) in x revolutions?

7. A cylindrical tank 7 feet in diameter contains water to a depth of 6 feet. Find the total area of the wetted surface.

8. Find the area of cardboard required for making a cylinder of radius 4 cm, height 6 cm, closed at both ends.

9. Find the volume of the following solid circular cylinders ($\pi = \frac{22}{7}$):

(*a*) Radius 3·5 feet, height = 5 feet.

(*b*) Radius 3·5 cm, height = 6 cm.

(*c*) Radius 3·5 cm, height = 2·3 cm.

(*d*) Diameter 1·4 metres, height = 1 cm.

(*e*) Diameter 1·4 dm, height = 1 cm.

10. Find the heights of the following solid circular cylinders:

(*a*) Volume 66 cu. cm, radius 2 cm.

(*b*) Volume 154 cu. cm, radius 3 cm.

(*c*) Volume 231 cu. in., diameter 5 inches.

(*d*) Volume 55 cu. ft, diameter 2·2 feet.

11. A circular cylinder has a volume of 44 cu. feet, and its height is 3·5 feet; find the diameter of its base.

12. A cylindrical wooden post is 2·52 metres long and 8 cm in diameter. Find the weight of the post if wood has a density of 0·7 gm per cc. Give your answer in kilograms.

13. A cylindrical well has a diameter of $3\frac{1}{2}$ feet. Find in inches the fall in the water level when a $2\frac{1}{2}$-gallon bucket is filled fourteen times from the well. 1 cu. feet of water = $6\frac{1}{4}$ gallons.

14. The iron cylinder of a garden roller is hollow and is filled with water. The internal radius of the cylinder is 10 inches and its internal length is 2 ft. Find the weight of the water inside the cylinder.

$\pi = 3{\cdot}142$. 1 cu. feet of water weighs 1000 ounces.

15. A pipe of internal diameter 2 inches delivers 15 gallons of water per minute. What is the speed of the water in feet per second if the pipe is full?

1 cu. feet of water = $6\frac{1}{4}$ gallons. $\pi = \frac{22}{7}$.

16. A pipe of internal diameter 3 inches delivers 45 gallons of water per minute. What is the speed of the water in inches per second, if the pipe is full?

17. Water flows into a rectangular cistern of cross section 11 sq. feet through a pipe with a circular cross-section of diameter 2 inches at the rate of 4 feet a second. At what rate per second will the water level in the cistern rise?

18. Water flows into a bath through a 4-inch pipe (i.e. internal diameter is 4 in.) at 5 feet per second. The bath is 50 ft long and 24 ft wide. How long will it take to fill it to a depth of 4 ft?

19. A cylindrical milk-measurer which holds a gallon is 5 inches deep. What is its radius?

20. The length of a column of mercury in a glass tube is 24·8 cm, and the weight of the mercury is 534·6 gm. If 1 cc. of mercury weighs 13·6 gm, find the average diameter of the tube.

THE LITRE

The litre is the capacity of a cube each side of which is 10 cm i.e. 1 dm.

$\therefore$ 1 litre = 1000 cc, also 1 litre = $1\frac{3}{4}$ pints.

EXERCISES 15

1. Find the capacity in litres of
   (*a*) a rectangular container with sides 12 cm by 10 cm by 8 cm.
   (*b*) a cubical container of side 25 cm.
   (*c*) a cylindrical vessel; $\left.\begin{aligned} r &= 7 \text{ cm.} \\ h &= 18 \text{ cm.} \end{aligned}\right\}$
   (*d*) a hemispherical bowl; $r = 12$ cm.

2. Find the gallon capacity of each of the containers of question 1.

3. A litre of wine is contained in a bottle of cross-section 48 sq. cm. How high is the bottle? (excluding the neck.)

4. A litre of wine is contained in a bottle of circular cross-section of radius 4·2 cm. How high is the bottle?

5. To what height would 15 litres of water fill a cylindrical container of diameter 12 cm?

THE RECTANGULAR WEDGE AND RIGHT RECTANGULAR PYRAMID

ABCDEF is a **rectangular wedge.** It is one-half of a rectangular solid cut across from AB to the opposite edge EF. Its volume is, therefore, $\frac{1}{2}$ area of base × height.

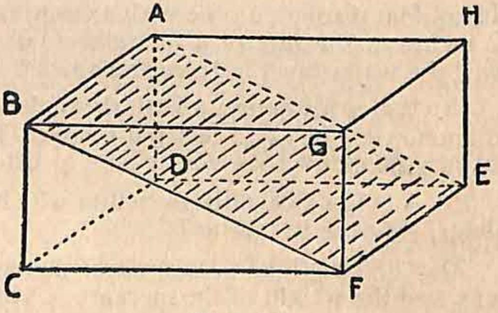

Fig. 10*a*

OABCD is a **right rectangular pyramid.** We have learnt that its volume $=\frac{1}{3}$ area of base $\times$ height. Its height OP is perpendicular to AC and BD and meets them where they cross.

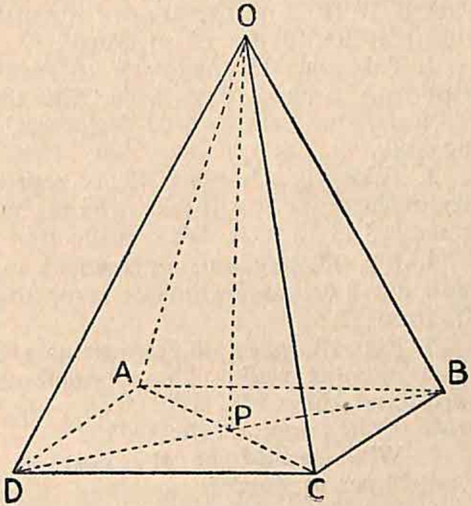

Fig. 10*b*

EXERCISES 16

1. In fig. 10*a* suppose ABFE $=20$ sq. ft; BF $=10$ ft; BC $=6$ ft. Find the volume of the wedge.

2. In fig. 10*a* if DCFE is a square, BC $=7''$ and the volume of the wedge $=100$ cub. in., find BE.

3. In fig. 10*a* if BCF is isosceles and ABCD is a square of area 60 sq. in., find the volume of the wedge.

4. In No. 1 of these Exercises find the superficial area of the solid wedge.

5. In No. 2 find the superficial area of the solid wedge.

6. In fig. 10*b*, if
   (*i*) AB $=6''$, BC $=8''$, OC $=12''$, find the volume.
   (*ii*) AB $=$ BC $=12''$, OC $=12''$, find the volume.
   (*iii*) AB $=$ BC $=12''$, OC $=20''$, find the volume.
   (*iv*) OP $=15''$, OC $=17''$, and ABCD is a square, find the volume.

AREA AND VOLUME

EXERCISES 17

1. Take fig. 11, page 45, to represent a pond surrounded by a gravel walk. The pond is $25\frac{1}{2}$ ft by $41\frac{1}{2}$ ft. The width of the

walk is 10 ft. Find the area of the walk and the cost of gravelling it at Rs 10 per 25 sq. yards.

2. Take fig. 13, page 45, to represent a garden bordered within by a walk 8 ft wide. The dimensions of the garden, including the walk, are 65 feet by $43\frac{1}{2}$ feet. Find the area of the walk.

3. Take fig. 13, page 45, to represent a photo frame. The size of the photo it will take is $8\frac{1}{2}$ in. by $5\frac{3}{4}$ in. The width of the frame is $\frac{3}{8}$ of an inch. What is the area of the frame ?

4. Fig. 18, page 46, represents a circular photo frame. The radii are 8·6 and 7·9 inches respectively. What is the area of the frame ?

5. Fig. 18, page 46, represents a circular pond surrounded by a circular walk. The circumference of the whole is 200 yards and the walk is 12 feet wide. What are the respective areas of the pond and the walk ?

What would be the cost of gravelling the walk at Re 0·25 per sq. yard ?

6. Fig. 12, page 45, represents a rectangular garden with two roads running across it. The roads are 11 feet wide and the dimensions of the garden are 100 yards by 62 yards. Find the area of the garden proper and the area of the total roadway.

7. In fig. 15, page 45, the centre of the small circle is A and the small circle passes through the centre, C, of the large circle. AC=7·5 inches. Find the area of the shaded portion.

8. In fig. 17, page 46, the centres of the three circles lie along the line AB. C is the centre of the big circle. AB=12·4 cm. Find the area of the shaded portion.

9. In fig. 19, page 46, the arcs are quarter-circles with A and D as centres respectively. The side of the square is 8·5 in. Find the area of the shaded portion.

10. In fig. 20, page 46, squares of side 3·3 in. are cut away from the corners of the rectangle. The dimensions of the rectangle are 12′ by 10′. Find the ratio of the area cut away to the area left.

11. In fig. 13, page 45, the sides of two squares are 7·3 and 8·5 in. respectively. Find the ratio of the shaded to the unshaded area.

12. In fig. 11, page 45, a square is inscribed in a circle. The side of the square is 4·7 cm. Find the ratio of the shaded to the unshaded area.

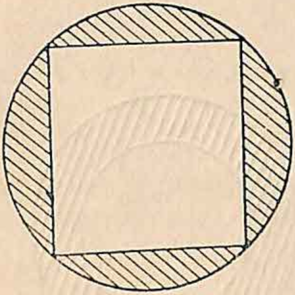

Fig. 11

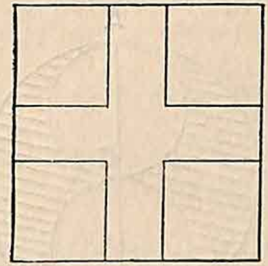

Fig. 12

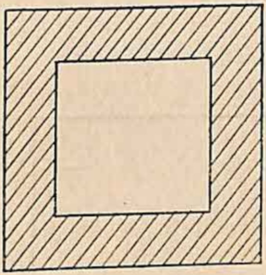

Fig. 13

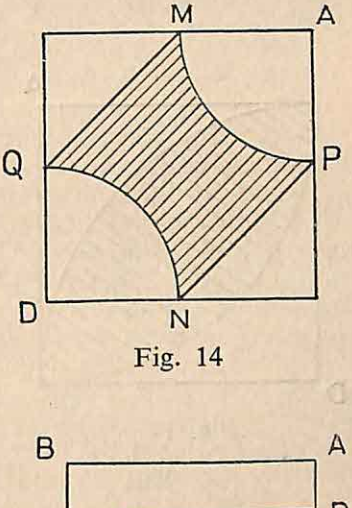

Fig. 14

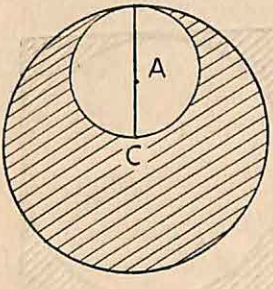

Fig. 15

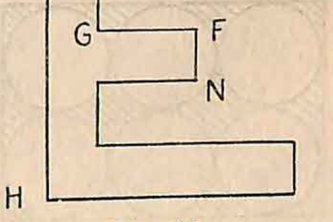

Fig. 16

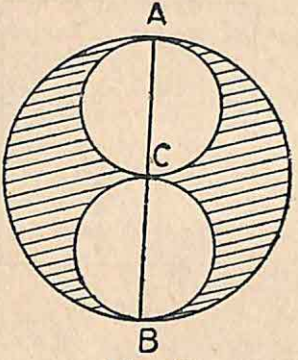

Fig. 17

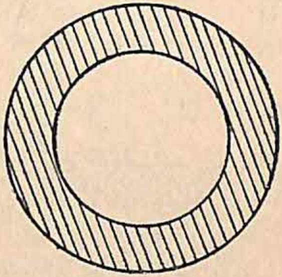

Fig. 18

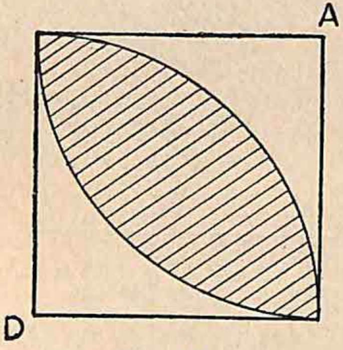

Fig. 19

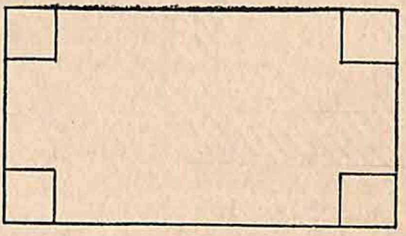

Fig. 20

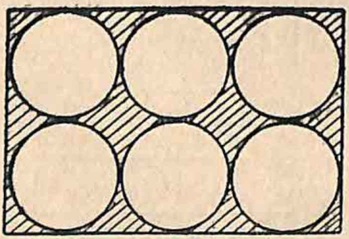

Fig. 21

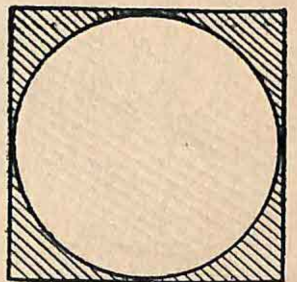

Fig. 22

13. In fig. 7 page 26, ABCD is a square. A, B, C, D are the centres of the four quarter-circles. AB=14″. Find the ratio of the shaded area to the rest.

14. Answer question 13, taking AB=21″. Why do you get the same answer as before ?

15. In fig. 22, page 46, a circle is inscribed in a square. The diameter of the circle is 10·5 ft. Find the ratio of the shaded area to the whole.

16. In fig. 18, page 46, the shaded part represents a ring cut out of a sheet of metal of uniform thickness. The radii are 3·1 cm and 4·5 cm. The metal weighs 0·56 gm to the square cm. What is the weight of the ring ?

17. Fig. 16, page 45, shows an E cut out from a square sheet of metal of uniform *thickness*. The measurements are BH=AB=6·23 in.; FN=AD=1·1 in.; GF=2·3 in. The weight of the metal is 3·5 lb. per sq. ft. What is the weight of the E to one decimal place?

18. In fig. 21, page 46, the circles are all equal, and the smaller side of the rectangle is 7·5 in. If the circles are stamped out, what is the weight of the part that remains, supposing the rectangle is made of material weighing 2·6 lb. to the sq. ft. ?

19. In fig. 14, page 45, the shaded portion is cut out from a square sheet of metal of side 11·5 in. A and D are the centres of the quarter-circles. M and N are the mid-points of the sides. What is the weight of the shaded portion if the metal sheeting weighs 2·3 lb. to the sq. ft ?

20. Fig. 12, page 45, represents a square sheet of metal of uniform thickness from which a cross of width 0·8 cm is cut out. The side of the square is 18·5 cm. Find the weight of the cross, if one sq. cm of the metal sheeting weighs 0·37 gm.

CHAPTER IX

VARIATION

The ratio as a Conversion Multiplier	—Alternation in a given ratio (increase or decrease)
Direct Variation	—Test for; Problems: rule "$\times \frac{\text{new}}{\text{old}}$"

Inverse Variation —Test for; Problems : rule "$\times\frac{\text{old}}{\text{new}}$"
Compensation for Faulty
Measures
Compound Variation

THE RATIO AS A CONVERSION MULTIPLIER

Example 1.

By what must 7 be multiplied to get 5 ?

$$7\times\tfrac{5}{7}=5.$$

Now, $7\times\frac{5}{7}=5$.

So $\frac{5}{7}$ is the required **conversion multiplier.**

EXERCISES 18

1. What conversion multiplier will change
   (*a*) 7 to 4 ? Ans. $\frac{4}{7}$
   (*b*) 8 to 5 ?
   (*c*) 8 to 6 ?
   (*d*) $7\frac{1}{2}$ to $5\frac{1}{4}$?
   (*e*) $7\frac{3}{4}$ to $9\frac{1}{2}$?
   (*f*) $3\frac{1}{5}$ to $4\frac{2}{5}$?
   (*g*) $11\frac{2}{8}$ to $3\frac{4}{9}$?
   (*h*) x to y ?
   (*i*) y to x ?
2. What conversion multiplier will change
   (*a*) Rs 11·50 to Rs 16·10 ?
   (*b*) Rs 20·80 to Rs 46·80 ?
   (*c*) £36 17*s*. to £43 11*s*. ?
   (*d*) £1517 5*s*. to £1083 15*s*. ?
   (*e*) 57 tons 6 cwt to 33 tons 8 cwt 2 qrs. ?
   (*f*) 109 tons 16 cwt 2 qrs. to 119 tons 7 cwt 2 qrs. ?
   (*g*) 13 hrs 20 min. to 8 hrs 20 min. ?
   (*h*) 4 miles 840 yds to 12 miles 550 yds ?
   (*i*) 12 ft 11 in. to 5 ft 2 in. ?
   (*j*) Area of a square of side 6 in. to area of a square of side 9 in. ?
   (*k*) The volume of a cube of side 8 in. to the volume of a cube of side 12 in. ?

We can compare two quantities of the same kind by means h eir ratio. Their ratio is the fraction which the first ty is of the second.

Thus the ratio of 3 ft to 4 ft 6 in. is $\frac{3}{4\frac{1}{2}}$ or $\frac{6}{9}$ or $\frac{2}{3}$

A ratio is best expressed in its simplest form. Remember that the ratio compares only quantities of the same kind, and, in forming the fraction, we must be careful to express both quantities in the same unit.

The ratio of 2 feet to 5 yards is not

$\frac{2}{5}$, but $\frac{2}{15}$.

We can, of course, compare quantities by stating the difference between them. In some cases a comparison by difference is more useful than a comparison by ratio.

If Harry is 12 years old and James 10 years,

"Harry is two years older than James," is a better way of comparing Harry's and James's ages than the statement—James's age is five sixths of Harry's age.

In this chapter and the next, however, we are going to be concerned with situations in which comparison by ratio is not only more useful, but necessary.

In the next chapter we shall treat the ratio as a mere comparison. In this chapter we shall make use of it as a conversion multiplier, that is, we shall use it as an *operator*.

The ratio of 3 to 8, or $\frac{3}{8}$, or 3 : 8 can be used as a conversion multiplier to covert 8 to 3.

The *ratio of a to b*, or $\frac{a}{b}$, or $a : b$ can be used as a conversion multiplier to change b to a;

for $b \times \frac{a}{b}$ gives a.

Copy down the examples of No. 2 (a to i) of the last exercises and write down the answers you obtained opposite each example.

In 2 (a) we found that $\frac{7}{5}$ (or 7 : 5) is the conversion multiplier that changes Rs 11·50 to Rs 16·10. Now study the following questions and their answers :

In what ratio must Rs 11·50 be altered to get Rs 16·10?
Answer 7 : 5.

In what ratio must Rs 11·50 be increased to get Rs 16·10 ? Answer 7 : 5.

(The second way of putting the question is not so good as the first. It is customary with some writers to use the phrases 'increase in a certain ratio' and 'decrease in a certain ratio'. It is better to say 'alter in a certain ratio', leaving it to be understood from the value of the ratio whether the alteration implies an increase or a decrease. Thus an alteration (or conversion) in the ratio 3 : 5 decreases the quantity altered, while an alteration in the ratio 5 : 3 increases the quantity.

Answer the following questions for each of Qs. 2(*a*) to 2(*i*) of the previous exercises.

(*i*) In what ratio must the first quantity be altered to get the second ?

(*ii*) What is the ratio of the second to the first ?

(*iii*) What is the ratio of the first quantity to the second ?

DIRECT VARIATION

	No. of Books	*No. of Rs*
(*i*)	3	6
(*ii*)	9	18
(*iii*)	18	36
(*iv*)	12	24
(*v*)	15	30
(*vi*)	27	54

@ Rs 2 a book.

Study the tables given. The numbers in the right-hand column give the number of *Rs* that will have to be paid for the number of *books* mentioned in the left-hand column.

What ratio are the *books** in the first row to the *books* in the fourth row ? They are in the ratio 1 : 4.

Notice that the corresponding numbers of *Rs* are also in the ratio 1 : 4.

Again, with the books in row (*iii*) and (*iv*): we can obtain the number of the *books* in row (*vi*) by altering the number in row (*iii*) in the ratio $\frac{27}{18}$ or $\frac{3}{2}$.

Notice that the number of *Rs* in row (*vi*) can be obtained from row (*iii*) by altering the number there in the ratio $\frac{54}{36}$, which is also $\frac{3}{2}$.

---

* Short for 'number of books'.

We shall arrive at a similar result with any other two rows. Test this out.

When two related quantities change in such a way that if one of them changes in a certain ratio, the other changes in the same ratio, one quantity is said to 'vary directly as the other.' Notice that **'variation'** here does not mean merely 'change', but a special kind of 'change'. In the tables, the *books* vary directly as the *Rs* and vice versa.

We can decide whether two quantities are such that **'one varies directly as the other'**, by considering whether one of them is doubled when the other is doubled, and whether it is trebled when the other is trebled, and so on.

For example:

(*a*) distance run and time taken by a train travelling at an uniform speed.

we argue shortly 'double the *time*, then double the *distance*.. ....' etc. And if this agreement fits in, then and then only does *distance* vary directly as *time*, and vice versa.

(*b*) temperature and time of the day.

At 3 a.m. the temperature is 54°

At 6 a.m. will it be twice 54°? This is not variation.

(*c*) In 16 strides a man covers 47 feet.

We argue 'twice the number of strides, twice the distance—' etc. Direct variation.

(*d*) At the age of 9 years a boy weighs 4½ stone. At 18 he will weigh——. Not variation.

In examples on direct variation we can always make use of the fact that the conversion multiplier or ratio will be the same for both quantities.

Example 2.

Eleven articles cost Rs 41·25.

What will 25 articles cost ?

First set out work like this :

Articles		Rs.	
11	cost	41·25	(old)
25	cost	?	(new)

The number of articles has been altered in the ratio.

$$\frac{25}{11} \qquad \frac{(\text{new})}{(\text{old})}$$

To obtain the new cost alter the old cost in the same ratio.

$$\begin{aligned}\text{Cost} &= \text{Rs } 41{\cdot}25 \times \tfrac{25}{11}\\ &= \underline{\text{Rs } 93{\cdot}75.} \qquad \text{Ans.}\end{aligned}$$

Example 3.

A train runs $107\frac{1}{4}$ km in $2\frac{3}{4}$ hours. How far will it go in the next $3\frac{1}{4}$ hours at the same rate ?

First set out the work:

km	*hours*	
$107\frac{1}{4}$	$2\frac{3}{4}$	(old)
?	$3\frac{1}{4}$	(new)

Alter $107\frac{1}{4}$ in the ratio $\dfrac{3\frac{1}{4}}{2\frac{3}{4}} \quad \left(\dfrac{\text{new}}{\text{old}}\right)$

Distance $=\frac{429}{4}\times\frac{13}{4}\times\frac{4}{11}$ km $=\frac{507}{4}=126\frac{3}{4}$ km.

Shortly,

km	*hours*
$107\frac{1}{4}$	$2\frac{3}{4}$
?	$3\frac{1}{4}$

$$\begin{aligned}\textit{The distance} &= 107\tfrac{1}{4}\times\frac{3\frac{1}{4}}{2\frac{3}{4}} \qquad \left(\frac{\text{new}}{\text{old}}\right)\\ &= \underline{126\tfrac{3}{4}\text{ km.}} \quad \text{Ans.}\end{aligned}$$

Example 4.

Thirty-two articles cost £76. How many articles can be bought for £49 17*s.* 6*d.* ?

First set out the sum:

articles	£	
32	76	old.
?	$49\frac{7}{8}$	new.

Alter 32 in the ratio, $\frac{\pounds 49\ 17s.\ 6d.}{\pounds 76}$ or $\frac{49\frac{7}{8}\ \text{(new)}}{76\ \text{(old)}}$

$$32\times\frac{49\frac{7}{8}}{76}$$
$$=\frac{32}{76}\times\frac{399}{8}$$
$=21$ *articles.*

17*s.* 6*d.* happens to be an 'easy' fraction of a £. Where we have not an 'easy' fraction, we may reduce to shillings or even to pence.

Shortly,

articles	£
32	76
?	$49\frac{7}{8}$

No. of *articles* $=32\times\frac{49\frac{7}{8}}{76}\quad\left(\frac{\text{new}}{\text{old}}\right)$

$=\underline{21}$. Ans.

Notice that in all the above examples we have altered the old value of one quantity in the ratio $\frac{\textbf{new}}{\textbf{old}}$ (values) of the other quantity.

We can always check the correctness of our ratio, or conversion multiplier, by considering whether it will have the effect of increasing or decreasing the quantity it multiplies.

In Ex. 2 above, we know from the question that our answer (the new cost) is to be greater than the old cost (Rs $41\frac{1}{4}$). So we could tell at once that it would be wrong to use the conversion ratio $\frac{11}{25}$.

Remember, however, to use this argument as a *check* only. The correct ratio for direct variation should be learnt as $\frac{\text{(new)}}{\text{(old)}}$ of the 'other' quantity.

EXERCISES 19

1. A journey of 432 km takes 6 days. How long will a journey of 720 miles take, if it is done at the same rate ?

2. In what time would an athlete, who can do the 100 yards flat race in $9\frac{4}{5}$ sec., do the 220 yards, were he able to run it at the same rate ?

3. Thirty-five baskets are required to carry 2205 oranges. How many such baskets will be needed to carry 6300 oranges ?

4. Sixty-nine cubic feet of a certain metal weigh 34860 lb. What will be the weight of 92 cubic feet ?

5. Two and a half gallons of paint are used to cover a surface 650 sq. ft in area. How much will be required for a surface of 1040 sq. ft ?

6. Five and a quarter cubic feet of a certain metal weigh 1563 lb. What will be the weight of $3\frac{1}{2}$ cubic feet ?

7. Twenty-two and a half metres of silk cost Rs 214·20. What will $6\frac{3}{4}$ m. cost ?

8. $11\frac{4}{11}$ acres of land are rented for Rs 250. For what should $10\frac{3}{4}$ acres be rented ?

9. Five-sevenths of a sum of money is £5 2*s*. 9*d*. How much will three and one-third times the same sum be ?

10. Fifteen steel trunks can be bought for Rs. 453·75. How many can be bought for Rs 514·25 ?

11. A carpet dealer finds that he can purchase 23 carpets of equal value for Rs 8050. How many carpets would he be able to purchase for Rs 12250 ?

12. On a transaction which brings in £92 12*s*. a man makes a commission of £17 7*s*. 3*d*. What would be his commission on a transaction which brought in £168 12*s*. ?

13. Fill in the blanks:

(*a*) Thirty-nine lamps cost Rs 1033·50
.... lamps cost Rs 1139·50 ?

(*b*) Rs 1968·75 will pay for 63 chairs.
Rs 2843·75 will pay for ... chairs ?

14. One hundred and fifty-nine bags of potato weigh 6 tons 19 cwt. 14 lb. How many bags will total a weight of 8 tons 19 cwt 1 qr. 14 lb. ?

15. A number of houses of equally long frontage completely occupy one side of a street. The distance between one end of the street and the farther boundary of the 25th house from that end is 639 yds. 1 ft 9 in. The street is 1381½ yards long. How many houses are there on the side in question ?

16. A clock gains 3 min. 20 sec. a day. It is set right at 6 p.m. on March 1st and not corrected when subsequently wound. When will it be showing right time again ?

17. It is found that a measure is contained exactly 21 times in a pail of capacity 6 gallons 3 quarts. How many such measures would be contained in a pail of capacity 11 gallons 1 quart ?

18. Seventeen men are engaged to build a wall 49 feet long. In 13 days they build 30⅓ ft. How long will they take to complete the work if they continue to build at the same rate ?

19. 17½ lb. of a certain metal cost £12 2*s*. 1*d*. What will be the cost of 49 lb. of another metal which is sold at double the price ?

20. Planks of wood of a certain thickness are sold at a price proportional to their width. If 19 planks of a certain width cost Rs 22·80, what will 33 planks of one and a half times the width cost ?

21. Five tables or twenty-two chairs cost Rs 575. What will three tables and fifty-five chairs cost ?

22. Six baskets of a certain kind or eleven boxes of another kind can just take 792 oranges. How many oranges in all can be carried in eight of each ?

23. A boy who has Rs 12·80 finds that with that money he can just buy either 6 lb. of drops or 4 lb. of toffee. He buys 1½ lb. of each. How much money has he left ?

24. The weekly wages of two men or three women or five boys are Rs 30. What will be the total weekly wages of five men, seven women and nine boys ?

25. A hockey stick costs two-fifths as much as a cricket bat. Three hockey sticks and two cricket bats cost Rs 64. What will have to be paid for two hockey sticks and three bats ?

26. A boy can work $\frac{3}{5}$ as fast as a man. The earnings for a certain period of three boys and four men, who are paid according to output, are Rs 58. What should be paid on the output of five boys and three men ?

27. The weight of 81·3 cub. in. of a certain metal is 21·95 lb. What is the weight of 40·3 cub. in. of the metal ?

28. An express train ran a distance of 176·8 km in 2·4 hours. If the speed was uniform throughout the journey, how long did the train take to do the first 40 km ?

29. If 35·7 metres of silk cost 335 fr. 58 cents, what will 72·5 metres cost ?

30. If 7·3 in., cut from a metal rod 16·5 in. long, weighs 83·6 gm., what will the rest of the rod weigh ?

31. If on an average the yield of paddy of 0·35 acre is 11480 kg, what should a field of 0·53 acre yield ?

32. A ladder resting against a wall is inclined in such a way that the seventh rung is 5·45 ft above the ground. The last rung is 32·7 ft above the ground. How many rungs has the ladder ?

33. When a man 6 ft tall walks 10·8 yds down an 'incline' his head is just on a level with the surface. How far below the surface will the point he reaches be when he has walked 9·4 yds further along the incline ?

34. Find the circumference of an arc which subtends an angle of 35° at the centre of a circle with a radius of 7 cm. ($\pi=\frac{22}{7}$). What is the area of the sector bounded by the arc ?

35. Answer question (34), given the following:—

(*a*) Radius=33 cm; angle=37°.
(*b*) Radius=8 cm; angle=72°.

(*c*) Radius $=12 \cdot 5$ cm; angle $=108°$.
(*d*) Radius $=5 \cdot 7$ cm; angle $=135°$.
$(\pi=\frac{22}{7})$.

36. Find the angle subtended at the centre by the arc in each of the following cases:

(*a*) Length of arc $=8 \cdot 5$ cm; radius $=7 \cdot 2$ cm.
(*b*) ,, of ,, $=13$ cm; radius $=5$ cm.
(*c*) ,, of ,, $=27 \cdot 5$ m; radius $=12 \cdot 2$ metres.
(*d*) ,, of ,, $=35\frac{1}{2}$ m; radius $=16$ m.

INVERSE VARIATION

Suppose we start with the assumption that twenty-four men can do a piece of work in six days; we can make the following tables:

	No. of Men	*No. of Days*
(i)	24	6
(ii)	12	12
(iii)	8	18
(iv)	4	36
(v)	48	3
(vi)	96	$1\frac{1}{2}$

To obtain row (ii) from row (i) we argue: 'Since half the number of men are working, they will take double the *time*.' To get row (iii) we argue 'Since one-third the number of *men* are working, they will take three times the *time*.' Argue out the other rows in the same way.

Next see rows (ii) and (iii). The *men* have been altered in the ratio $\frac{8}{12}$ or $\frac{2}{3}$. The *days* have been altered in the ratio $\frac{18}{12}$ or $\frac{3}{2}$, which is the inverse of $\frac{2}{3}$.

We can now put the argument as follows: The *days* are altered in inverse ratio to the *men*.

When two related quantities change in such a way that if one of them alters in a certain ratio the other alters in the inverse ratio, the two quantities are said to **vary inversely as each other** or to be in **inverse variation.**

In **direct variation** we used the conversion ratio $\frac{\textbf{new}}{\textbf{old}}$.

When we work examples in **inverse variation** we must remember to use the ratio $\frac{\textbf{old}}{\textbf{new}}$.

(We should also use the additional check of considering whether the ratio we use has the effect of increasing or decreasing the quantity it multiplies.)

Example 5.

If there is enough food to last a party of 30 men 15 days, how many men will be able to manage in the same way on the same quantity of food for 25 days ?

	days	*men*
old	15	30
new	25	?

$$\text{No. of } \textit{men} = 30 \times \tfrac{15}{25} \ldots \left(\frac{\text{old}}{\text{new}}\right)$$

$$= \underline{18}. \text{ Ans.}$$

Example 6.

A tap which supplies water at 8 gallons an hour can fill a cistern in $12\frac{1}{2}$ hours. How long will a tap take which can supply $6\frac{1}{4}$ gallons an hour ?

	gallons	*hours*
old	8	$12\frac{1}{2}$
new	$6\frac{1}{4}$	?

$$\text{No. of } \textit{hours} = 12\tfrac{1}{2} \times \frac{\text{old}}{\text{new}}$$

$$= \frac{25}{2} \times \frac{8}{6\frac{1}{4}}$$

$$= \underline{16 \text{ hours.}} \text{ Ans.}$$

EXERCISES 20

1. A train can do a certain journey in 7 hours if it travels at 36 km an hour. How long will it take if it travels at 42 km an hour ?

2. Twenty articles can be bought for a certain sum when the price of each article is Re 0·81. How many articles can be bought for the same sum when the price rises to Re 0·90 ?

3. Of two fitting cog-wheels, one has 25 teeth and the other 45. How many revolutions will the second wheel make to every 81 revolutions of the first wheel ?

4. A tap which supplies water at 48 litres an hour can fill a cistern in $7\frac{1}{2}$ hours. How long will it take to fill the cistern if it supplies water at 72 litres an hour ?

5. Twelve loaves can be bought for a rupee when the price of flour is Rs 10 a bag. How many loaves can be bought for a rupee, when the price is Rs 8 a bag?

6. It is found that a book will contain 350 pages if 32 lines are allowed to a page. How many lines should be allowed to a page if the book is to contain only 280 pages ?

7. A garrison of 2000 has food enough to last them 45 days. After 12 days 350 men leave. How much longer will the food last ?

8. A garrison of 1500 men have enough food to last them 47 days. After 13 days they are reinforced by 200 men. How many days will the food now last ?

9. Thirty-five articles can be bought for a sum of money. How many articles can be bought for the same sum if the price of each article is lowered in the ratio $\frac{7}{9}$?

10. A party of men has food enough to last them 33 days. If each man's ration is increased in the ratio $\frac{11}{9}$, how long will the food last ?

11. Fifteen men can do a piece of work in 7 days. How long will they take if they speed up their work in the ratio $\frac{7}{5}$?

12. 7 men or 9 women can do a piece of work in 32 days. How long will 7 men and 9 women take to do the same piece of work ?

13. Three men or five boys can do a piece of work in 17 days. How long will five men and three boys take ?

14. Three men and five boys can do a piece of work in 18 days. How long will ten men and three boys take to do it, if a man can work one and a half times as fast as a boy ?

15. Thirty men are engaged to do a piece of work which they can do in 8 days. After 2 days ten men are taken off. How long will the remaining 20 take to complete the work ?

16. Twenty-five men start on a piece of work which they can do in 12 days. After 3 days, 5 more men are put on. How much sooner will the work be done ?

17. Thirty-six men can do a piece of work in 12 days. After they have worked for 6 days at it, 9 men are taken off. How much longer will the work take ?

COMPOUND VARIATION

Example 7.

The carriage charge for 15 tons of material for 200 miles is Rs. 320. What is the charge for carrying 25 tons 150 miles at the same tariff ?

First set out the problem as follows:

tons	*miles*	*Rs*	
15	200	320	(old)
25	150	?	(new)

Here the *charge*, Rs 320, has to be altered for two reasons;
(i) the *weight* carried has been changed.
(ii) the *distance* has been changed.

Notice that the charge (Rs) (i) varies directly as the *distance*.
(ii) varies directly as the *weight* carried.

Consider these in succession.

First alter for the *miles*.

The *charge* becomes Rs $320 \times \frac{150}{200} \frac{(\text{new})}{(\text{old})}$;

(do not simplify).

Then alter this again for the tons.

$$\text{The } charge = \text{Rs } 320 \times \tfrac{150}{200} \times \tfrac{25}{15} \frac{(\text{new})}{(\text{old})}$$

$$= \text{Rs } 400.$$

We can show this all in one step thus:

$$\text{The } charge = \text{Rs } 320 \times \tfrac{150}{200} \times \tfrac{25}{15} = \underline{\text{Rs } 400.}\ \text{Ans.}$$

Example 8.

Twenty-one men can dig a trench 70 feet long in 6 days. How long will it take 30 men to dig a trench 50 feet long, of the same depth and width?

First set out the problem:

men	*feet*	*days*	
21	70	6	(old)
30	50	?	(new)

We alter first for the *feet* ... direct variation. (Why?)
We alter next for the *men* ... inverse variation. (Why?)
In one step:

$$\text{The answer} = 6 \times \tfrac{50}{70} \left(\frac{\text{new}}{\text{old}}\right) \times \tfrac{21}{30} \left(\frac{\text{old}}{\text{new}}\right) \text{ days}$$

$$= \underline{3 \text{ days.}}\ \text{Ans.}$$

Example 9.

Sixteen men working 9 hours a day can dig a trench 72 feet long, 24 inches wide and 1 foot deep, in 6 days. How many men working 7 hours a day will be needed to dig a trench 80 feet long, $2\frac{1}{2}$ feet wide and 14 inches deep in 8 days?

First set out the problem as follows :

men	*hours a day*	*length* (ft.)	*width* (ft.)	*depth* (in.)	*days*	
16	9	72	2	12	6	(old)
?	7	80	$2\frac{1}{2}$	14	8	(new)

$$\text{No. of men} = 16\times\frac{9}{7}\times\frac{80}{72}\times\frac{2\frac{1}{2}}{2}\times\frac{14}{12}\times\frac{6}{8}=\underline{25}.\ \text{Ans.}$$

EXERCISES 21

1. The rent for 8 acres of land for 9 months is Rs 135. What is the rent for 9 acres for 8 months?

2. In how many days of 8 hours each, can 12 men do the same amount of work as 10 men working 9 hours a day can do in 16 days?

3. The rent payable for grazing 16 cows for 12 days on a piece of land is Rs 12. What is the rent payable for grazing 20 cows for 8 days?

4. Fifteen chests of tea, each chest containing 20 kg cost Rs 350. What will be the price, at this rate, of twelve chests, each containing 25 kg?

5. If 27 rectangles of length 32 cm and breadth $18\frac{1}{3}$ cm are exactly equivalent to a certain area, how many rectangles of length 24 cm and breadth 11 cm will also equal this area?

6. Twenty-seven kg of metal will fit into a rectangular box with sides 11″ by 9″ by 7″. How many kg will fit into a box 7″ by 6″ by 2″?

7. Twenty-seven boxes 12″ by 9″ by 7″ will exactly fit into a certain space. How many boxes 7″ by 6″ by 2″ will fit into a space of the same volume?

8. Three pumps working 8 hours a day can empty a tank in 2 days. How many hours a day must four pumps work to empty the tank in 1 day?

9. If 23 men earn Rs 632·50 in five days, how much will 31 men earn in 22 days?

10. The carriage for 3 tons 5 cwt of material for 327 miles is Rs 218. What will be the freight for 4 tons 11 cwt for 405 miles?

11. The carriage for a certain weight of mineral for a cer-

tain distance is Rs 400. What is the carriage for three-quarters that weight of mineral for three-fifths of the distance?

12. Twenty men can do a piece of work in 9 days, working 6 hours a day. How many men would be able to do the same piece of work in 10 days, working 5 hours a day, if five of the latter set can do as much work as six of the former in the same time?

13. Twenty-two men working six hours a day can dig a trench 8 yards long, 11 feet wide and 14 inches deep in a certain time. How many men working 9 hours a day will be needed to dig a trench 7 yards long, 9 feet wide and 1 foot deep in the same time?

14. Twenty-two men working six hours a day can dig a trench 70 yards long, 11 feet wide and 13 inches deep in 5 days. How many days will 25 men, working 4 hours a day, take to dig a trench 50 yards long, 13 feet wide and 14 inches deep?

15. Fifteen men working six hours a day can dig a trench of a certain length, width and depth in 6 days. How many men working at 9 hours a day will be needed to dig a trench twice as long, one and a half times the width and three-quarters the depth in five days?

CHAPTER X

RATIO : SHARING : SCALE : PERCENTAGE COMPOSITION : PARTNERSHIP

Sharing

(*i*) *To divide a number in the ratio 3 : 4, we first divide it into 7 equal parts and then take 3 parts for one share and 4 parts for the other share.*

To divide a number in the ratio a : b, we first divide it into a+b equal parts and then take a parts for one share and b parts for the other share.

(*ii*) *To divide a number in the ratio* $\frac{1}{2} : \frac{1}{5}$, *we first clear denominators so as to get whole numbers.*

The ratio is now 5:2. We then proceed as in (*i*).

(*iii*) *To divide a number into three shares in the ratio 3 : 4 : 8, we first divide it up into (3+4+8) or 15 equal parts. We then allot 3, 4 and 8 of these parts as the respective shares.*

Example 1.

Divide Rs 180 into three shares proportional to 3, 4 and 8.

Divide Rs 180 into 15 equal parts.

Each part = Rs 12.

The shares are Rs $12 \times 3 =$ Rs 36,
Rs $12 \times 4 =$ Rs 48,
and Rs $12 \times 8 =$ Rs 96. Ans.

Example 2.

Divide £186 among A, B and C in such a way that A will get $\frac{4}{5}$ of B's share, and B will get $\frac{3}{7}$ of C's share.

$$\begin{array}{lccc} \text{We write} & A & B & C \\ & 4\ : & 5 & \\ & & 3\ : & 7 \end{array}$$

If we multiply the top line by 3 and the lower line by 5, B will be 15 in both cases and we can compare all three.

The ratio then becomes—12 : 15 : 35.

We then divide £186 by (12+15+35).

£ $\frac{186}{62}$ = £3.

A gets £12×3; B gets £15×3; C gets £35×3. Ans.

EXERCISES 22

1. Divide 104 into three parts, proportional to 3, 4 and 6.
2. Divide 1734 into three parts, proportional to 4, 6 and 7.
3. Divide 583 into three parts, so that the first part is equal to the second and each of them bears to the third the ratio 3 : 5.
4. The sum of four numbers is 82. They are proportional to 8, 9, 11, 13 respectively. What are the numbers?
5. Divide 72 into three parts in the ratio 5 : 6 : 7.
6. Divide 72 into two parts in the ratio $\frac{1}{2} : \frac{1}{3}$.
7. Divide 104 into three parts in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$.
8. Divide 536 into three parts in the ratio $\frac{1}{5} : \frac{2}{3} : \frac{1}{4}$.
9. Divide 2173 into two parts proportional to $4\frac{1}{2}$ and $5\frac{3}{4}$.
10. The sum of three numbers is 2947. They are proportional to $3\frac{1}{2}$, $4\frac{1}{5}$, $6\frac{1}{3}$ respectively. Find them.
11. A piece of land is to be divided between two men in the ratio 5 : 9. What fraction of the whole does the second lose if the land is divided instead in the ratio 9 : 5?

12. A piece of land is to be divided between two men in the ratio 5 : 7. What fraction of the whole does the first man lose if it is divided instead in the ratio 5 : 8?

13. Divide 735 into three parts proportional to 4, 5 and 7.

14. Divide 735 into three parts proportional to $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{7}$.

15. Two numbers, one of which is 12 greater than the other, are in the ratio 7 : 9. Find them.

16. The angles of a triangle are in the ratio 7 : 6 : 8. Find the largest angle.

17. The angles of a quadrilateral are in the ratio, $\frac{1}{2} : \frac{1}{4} : \frac{1}{3} : \frac{1}{5}$. Find them.

18. Divide 4·6 in the ratio 1·2 : 2·3 : 0·5.

19. Divide 87·6 in the ratio 3·7 : 2·23 : 1·3.

20. Divide a line 531·1 cm long into two parts proportional to 3·5 and 7·8.

21. A line is divided into two parts proportional to 5·6 and 1·3. The smaller part is 48·1 cm long. How long is the line?

22. The sides of a triangle are proportional to the sides of another triangle. The perimeter of the first triangle is 11·2 cm. The sides of the second triangle are 3·1 cm., 2·5 cm. and 2·8 cm. What are the sides of the first triangle?

23. Divide Rs. 2681 among three people, giving the first twice as much as the second, but half as much as the third.

24. Divide Rs 97·20 among three people, giving the first three times as much as the second, and the third as much as the other two together.

25. Three men have to share Rs 276·25 in such a way that the second will receive a third of the share of the first, and the third a quarter of the share of the second. How much more will the first man receive than the third?

26. Four persons have to share Rs 209·34 in such a way that the first and second will get the same amount, the third and fourth the same, and each of the former will get $\frac{4}{5}$ of the share of each of the latter. What will each person get?

27. How must an allotment of $1599\frac{1}{2}$ acres of land be divided among three persons so that their shares will be in proportion to $7\frac{1}{2}$, $3\frac{1}{3}$ and $4\frac{2}{5}$ respectively?

28. Divide 66 into three parts such that the first will be $\frac{3}{4}$ of the second, and the second $\frac{4}{5}$ of the third.

29. Divide 1545 into three parts such that the first will be $\frac{3}{5}$ of the second and the second $\frac{6}{11}$ of the third.

30. If it is agreed that A will receive $\frac{3}{7}$ of B's share and B $\frac{2}{5}$ of C's share, how will they divide £ 803 between them?

31. If it is agreed that A will receive $\frac{3}{7}$ of B's share and C $\frac{2}{5}$ of A's share, how will they divide Rs 1722 between them?

32. Divide Rs 463·50 among A, B, and C in such a way that the ratio of A's share to B's is 7 : 9, and B's share to C's is 12 : 13.

SCALE

The Scale of a map is the ratio

$$= \frac{\textbf{distance on map}}{\textbf{corresponding actual distance}}.$$

This ratio expressed in the form $\frac{l}{m}$ is called the **representative fraction** of the map.

EXERCISES 23

1. A map is drawn to the scale, 2·5 feet to 100 miles. Find the representative fraction of the map.

2·5 feet represents 100 × 5280 feet.

$$\text{The scale } = \frac{2{\cdot}5}{100 \times 5280}$$

$$\text{Rep. fraction (R.F.)} = \frac{1}{\frac{100 \times 5280}{2{\cdot}5}} = \underline{1 : 211200.} \text{ Ans.}$$

2. Find the R.F.s of the maps that have been drawn to the following scales:

(*a*) 5 inches to the furlong.
(*b*) 5 feet to the mile.
(*c*) 6 inches to the mile.
(*d*) 5 cm to the km.
(*e*) 5 inches = ½ mile.
(*f*) 1 foot = 5 miles.
(*g*) 1 cm. = 15 km.

(*h*) 6 feet = 14 miles.
(*i*) 6·5 ft = $7\frac{1}{4}$ furlongs.
(*j*) 0·1 inch = 1 mile.
(*k*) $\frac{1}{2}$ inch = $\frac{1}{3}$ mile.
(*l*) $\frac{3}{4}$ inch = $\frac{4}{5}$ miles.

3. What acreage is represented by an area of 3 sq. in. on a map drawn on the scale, 2 feet to the mile?

4 sq. ft. represents 1 sq. mile.
i.e., 4×144 sq. *in.* ,, 640 acres.
3 sq. in. ,, ?

$$1\times\frac{3}{4\times144}\times640 \text{ acres}=\underline{3\tfrac{1}{3}\text{ acres.}}\text{ Ans.}$$

4. The R. F. of a map is $\frac{1}{2640}$. What length does an inch on the map represent?

What length does a foot on the map represent?

By what length on the map will 5 miles be represented?

By what length will $7\frac{1}{3}$ miles be represented?

5. A map is drawn to the scale $\frac{1}{2}$ inch = 47 miles. What length does $3\frac{1}{4}$ inches on the map represent? By what length on the map will $70\frac{1}{2}$ miles be represented?

6. It is known that the actual distance between two places, A and B, on a map, is $5\frac{1}{2}$ miles. The distance between them on the map is 4 inches. What is the actual distance between two places $2\frac{1}{2}$ inches apart on the map?

7. The actual distance between two places is $7\frac{1}{4}$ miles. Their separation on the map is $2\frac{1}{2}$ inches. What would be the separation on the map of two places 10 miles apart?

8. The scale of a map is 1 cm=10 km. What area is represented by 4 sq. cm. on the map?

9. The R. F. of a map is 1 : 2500. What area does a sq. in. on the map represent?

10. The scale of a map is 2·5 in. to the mile. What area (in sq. miles) does a rectangular bit on the map 2·3 in. by 4·1 in. represent?

11. The scale of a map is 3 in. to 10 miles. What area will 5 sq. inches on the map represent?

12. The scale of a map is 1·5 feet to 120 miles. What area will 10 sq. inches on the map represent?

13. The R. F. of a map is $\frac{1}{150,000}$. What area does $2\frac{1}{4}$ sq. inches on the map represent?

14. The scale of a map is 3·5″ to the mile. What area does a rectangular bit on the map 4·2″ by 4·9″ represent?

15. Two maps are drawn of the same place. The R. F. of one is 1 : 25000 and the R. F. of the other is 1 : 36000. What is the ratio of the size of the first map to the size of the second?

16. Two equal-sized maps are drawn of two different square fields. The R.F.s of the maps are respectively 1 : 34,000 and 1 : 51,000. Which field is bigger and what is the ratio of their acres?

COMPOSITION

EXERCISES 24

(*a*) Epsom Salts (without associated water).

Magnesium. by weight.	... 24 parts
Sulphur.	... 32 parts
Oxygen.	... 64 parts

(*b*) An Indelible Ink.

Aniline oil	... 87 parts.
Potassium Chlorate.	... 5 parts.
Distilled Water.	... 45 parts.
Hydrochloric Acid.	... 67 parts.
Copper Chloride.	... 6 parts.

(*c*) Cleaner.

Acetic Ether.	... 25 parts.
Green Soap.	... 22 parts.
Liquid Ammonia.	... 33 parts.
Boiling Water	... 700 parts.

(*d*) German Silver.

Copper.	... 55 parts.
Zinc	... 23 parts.
Nickel	... 15 parts.

(*e*) Mr. Smith's Orchard.

Apple Trees.	... 270.
Orange Trees.	... 320.
Other Trees.	... 251.

(*f*) Railway Earnings during a certain period.

Goods Service.	Rs. 725,600.
Passenger Service.	Rs. 341,250.
Bus.	Rs. 79,400.
Restaurant ,,	Rs. 8,000.

Answer the following questions:

1. In (*a*) find the weight of magnesium in 350 gm of Epson Salts.

2. In (*b*) give the percentage composition of indelible ink.

3. In (*b*), if just 15 gm of copper chloride are available and are to be used, how much potassium chlorate will be required?

4. In (*c*) what percentage of the whole is the quantity of green soap to be used?

If 63 gm of ammonia are used, how much boiling water will be required?

5. In (*d*) find the percentage composition of the alloy.

6. In (*d*) find how much of each metal there is in 3255 kg of the alloy.

7. In (*d*), if 12 kg of copper are to be used find (approximately) what quantities of the other metals should be used.

8. In (*e*) find the percentage distribution of trees in the garden.

If Mr. Smith valued an orange tree at $\frac{2}{3}$ an apple tree and each of the 'other varieties' at $\frac{1}{2}$ an orange tree, set out the relative values of apple trees : orange trees : other trees.

9. In (*e*), if the owner cuts down 10% of his apple trees and 20% of his orange trees, what will the percentage distribution of the trees then be?

10. In (*f*), find what percentage of the total earnings the goods service brings in.

11. If in (*f*) all the services, other than the goods service, earn 50% more, what percentage of the whole will the goods earnings represent?

12. In (*f*), if the earnings from the passenger service fall off by $33\frac{1}{3}\%$, while the rest of the services earn the same, by how much will the combined percentage earnings of the other services go up?

EXERCISES 25

(In the following examples it is assumed that the Partners have agreed to divide profits in the ratio of their respective capitals.)

1. Two men go into partnership, one putting in a capital of Rs 7500 and the other a capital of Rs 5000 into the business. They agree to divide all profits in proportion to their capital. How should they divide an annual profit of Rs 1,062·50 between them. What percentage profit does each make?

2. Continuing with question (1), if in the following year the first man receives Rs 168·75 as his share of the profit for the year, what does the second man receive?

3. Three partners have capitals of 32500 frs.; 61300 frs. and 42000 frs. in a business. How should they divide a profit of 15277 frs. 50 cents between them?

What percentage profit does each man make?

4. Three men go into partnership with capitals of $40000; $65000 and $78000 respectively. They come to an agreement that the first will manage the business on condition that he gets a third of the profits, whatever they are, while the other two divide the remainder in the ratio of their capitals. How should they divide up a profit of $4826 25c. among them?

5. Of three partners, whose capitals are in the ratio 5 : 6 : 9· the first's share of a year's profits works out at £601 7*s*. 6*d*, What are the total profits and the other partners' shares?

6. Fill in the blanks in the following. The numbers give the capital subscribed by three partners in a business and their first two years' profit.

Partners	Capital	Profits: 1st year	Profits: 2nd year
A	£7200	£482 12*s*. 0*d*	
B		£301 12*s*. 6*d*.	
C	£5400		
Totals			£1466 11*s*. 3*d*.

7. Fill in the blanks in the following:

Partners	Capital	Profits: 1st year	Profits: 2nd year
A	Rs 54000		
B		Rs 5125	
C	Rs 63000	Rs 4305	
Totals			Rs 18432

8. Fill in the blanks in the following:

Partners	Capital	Profits: 1st year	Profits: 2nd year
A		Rs 9450	Rs 12600
B		Rs 5700	
C		Rs 7350	
Totals	Rs 150000		

CHAPTER XI

(i) CONVERSION CONSTANTS AND THEIR RECIPROCALS

(ii) RATE OF EXCHANGE

1 metre	**(100 cm)**	**= 39·37 inches**	**(approx.)**
1 are	**(100 centares)**	**= 119·6 sq. yds**	**(,,)**
1 litre		**= $1\frac{3}{4}$ pints**	**(,,)**
1 kilogram		**= 2·2 lb.**	**(,,)**
1 cub. ft holds		**6·25 gallons of pure water**	**(,,)**

Numbers, like those on the right hand side of the above equivalents, are sometimes known as '**conversion constants**'. They give us the relationship between units belonging to different systems of measurements of the same kind.

In each case multiplication by the proper constant will convert the measurement from one in the unit specified on the left-hand side to its equivalent in the unit specified on the right-hand side.

Thus: if we know a certain capacity in *litres*, we have only to multiply the number of litres by $1\frac{3}{4}$ and we have the same capacity measured, instead, in *pints*.

To convert from the right-hand units to the left we must divide by the proper constant; but, if the constant is a decimal quantity, it will most often be more convenient to multiply, instead, by the reciprocal. The reciprocal may either be memorized or obtained from a table of reciprocals.

Look up a table of reciprocals for the reciprocals of the above constants and verify the following (approximate) equivalents.

1 inch =2·54 cm
1 sq. yd =0·0084 are.
1 lb. =0·454 kg
1 gallon of pure water =0·16 cub. ft

We may add to our tables

$$\mathbf{1\ pt = \frac{4}{7}\ litre}$$

and the diameter of a circle=·3183×circumference.

Example 1.

How many kg are there in 5·6 lb. ?

5·6×0·454 kg
=2·54 kg.

Example 2.

What is the cubical capacity of a tank that can hold 158·6 gallons ?

158·5 gal. =158·6×0·16 cub. ft
=25·376 cub. ft.

Example 3.

Verify, without actual division and without consulting tables, that 0·3183 is approximately the reciprocal of π (=3·142).

3·142×0·3183
=1·0000986.

Since the product is very nearly 1, the numbers are very nearly reciprocals.

Note:

$$\text{If } a = \frac{1}{b}$$

$$\text{then } ab = \frac{1}{1}$$

$$\text{and } b = \frac{1}{a}.$$

We can avoid division by a decimal quantity by multiplying, instead, by its reciprocal.

Example 4.

Simplify $\dfrac{125}{14{\cdot}29}$.

$$\frac{125}{14{\cdot}29} = 125 \times 0{\cdot}07 = \underline{8{\cdot}75}.\ \text{Ans.}$$

EXERCISES 26

(*Use reciprocals in order to avoid division.*)

1. (*a*) 16 in. = cm.

(*b*) 25 pints = litres.

(*c*) 30 lb. = kg.

(*d*) 75 gallons = cubic ft.

(*e*) 32 feet = metres.

(*f*) 50 quarts = litres.

(*g*) 14 stone = kg.

(*h*) 320 quarts = cubic ft.

(*i*) The circumference of a circle is 72, the diameter is——

(*j*) The circumference of a circle is 11·6, the diameter is——

(*k*) 7·3 pints = litres.

(*l*) 50·7 lb. = kg.

(*m*) 73·8 gallons = cubic ft.

(*n*) 13·6 stone = kg.

(*o*) The circumference of a circle is 82·5″; the diametre is——

(*p*) A tin box 7′×6′×5′ holds———gallons.

(*q*) It will hold———litres.

(*r*) To divide by 73·6, multiply by———

(*s*) To divide by 0·064, multiply by———

(*t*) To divide by 83·1, multiply by———

2. Take the answers of questions 1, *a-j*, and reconvert from the right hand to the left. See that you get back (approximately) the quantities on the left.

3. Evaluate (with the aid of tables of reciprocals)

$$\frac{10}{7{\cdot}132};\ \frac{100}{83{\cdot}54};\ \frac{15}{6{\cdot}27};\ \frac{25}{73{\cdot}6};\ \frac{30}{92{\cdot}6};\ \frac{88}{61{\cdot}5};\ \frac{11}{25{\cdot}7};\ \frac{13}{71{\cdot}9}.$$

4. Test, without actual division and without consulting tables of reciprocals, whether or not the following pairs of numbers are (approximately) reciprocals.

a. 31·2, ·032 *b.* 4·23, ·294 *c.* 5·33, ·1876

d. 53·4, ·0203 *e.* 677, ·001477 *f.* ·789, 1·267

RATE OF EXCHANGE

The rate at which the currency of one country has to be converted to find its equivalent value in terms of the currency of another country is called a **'rate of exchange'.**

There are, for example, the London-Paris rate of exchange, and the London-New York rate of exchange. These and other rates of exchange fluctuate for reasons that you will learn elsewhere. They can always be looked up in the daily newspaper where they are printed under the heading 'Exchange Rates'.

The rate for conversion between the £ and the rupee, however, is fixed for the present by statute at 1*s*. 6*d*. to the Rupee. This is called the London on Bombay rate.

EXERCISES 27

1. Convert the following sums to Indian money at the rate of 1*s*. 6*d*. = Re 1 (or 6*d*. = Re $\frac{1}{3}$):—

(*a*) 3*s*. 6*d*.
= 7 six-pences
= $7 \times$ Rs. $\frac{1}{3}$
= Rs $\frac{7}{3}$
= $\underline{\text{Rs } 2{\cdot}33}$
(to the nearest nP.). Ans.

(*b*) 4*s*. 9*d*.
= $9\frac{1}{2}$ six-pences
= $9\frac{1}{2} \times$ Re $\frac{1}{3}$
= Rs $\frac{19}{6}$
= $\underline{\text{Rs } 3{\cdot}17}$
(to the nearest nP.). Ans.

(*c*) 6*s*. (*d*) 17*s*. 6*d*. (*e*) £1. (*f*) £1 6*s*.
(*g*) £1 10*s*. (*h*) £5 6*s*. (*i*) 9*d*. (*j*) 1*s*. $1\frac{1}{2}$*d*.
(*k*) £3 7*s*. 6*d*. (*l*) £5. (*m*) £5 10*s*. (*n*) £7 17*s*. 6*d*.
(*o*) £100. (*p*) £1,000. (*q*) £1,000,000 (give your answer in lakhs).

2. Convert the following sums to £ *s*. *d*. at the rate of 1*s*. 6*d*. to the Rupee:

50 nP. = 9*d*.
6 nP. = $\frac{9}{8}$*d*. (approx).

(*a*) Example:
Rs 3·75
$= 3{\cdot}75 \times 1\frac{1}{2}$*s*.
$= 5{\cdot}625$*s*.
= $\underline{5s.\ 7\frac{1}{2}d.}$ Ans.

(*b*) Rs 7·75 (*c*) Rs 10 (*d*) Rs 15
(*e*) Rs 20 (*f*) Rs 50 (*g*) 100
(*h*) Rs 160·50 (*i*) Rs 175·75 (*j*) Rs. 250
(*k*) Rs 375·75 (*l*) Rs 1,000 (*m*) Rs. 1,00,000(lakh).

CHAPTER XII

RATES

(*i*) Compound Units. A rate is a compound unit.
$\text{Rate} = \dfrac{\text{Unit of one kind}}{\text{Unit of another kind}}$. Speed is one kind of rate.

'(*ii*) The rate as a conversion multiplier.

(*iii*) Change the unit of rate from one system to another.

Distance. (feet)	0	12	20	32	36	60
Time (seconds)	0	3	5	8	9	15

The above table gives the *distance* (feet) travelled by a body in a given *time* (seconds).

If we divide the top quantity by the lower quantity we get 4 in each case. Now what name are we to give this 4 ? It must be 4 of some unit. What unit ?

Of course, we all know that 4 represents the speed of the body. It does help a great deal to say that the 4 is 4 units of speed; but it does not help completely; for if the time were in hours instead of seconds, we would still get 4 on division, and yet the two speeds are surely not the same.

We can meet the difficulty by writing 4 feet per sec. The unit is now a 'foot per sec.'. This is the unit of speed that has been used, and the speed in question is 4 of these units.

Thus, also, in 4 'miles per minute', a 'mile per minute' is the unit of speed.

We shall see presently that there is a great advantage in writing these units thus:

$4\,\frac{\text{foot}}{\text{second}}$; or, in the second case, $4\,\frac{\text{mile}}{\text{minute}}$.

This way of writing the unit shows clearly *how* the unit of the new kind is 'made up'. *In this case*, by division of units of two kinds of quantities.

Thus: 1 foot is a unit quantity of length.

1 second is a unit quantity of time.

$1\,\frac{\text{foot}}{\text{second}}$ is a unit quantity of speed.

When a unit is 'made up' by multiplication or division of two or more other kinds of units, we call it a **compound unit.**

Thus a unit of momentum is a
lb. × sec.
In our case a unit of speed is a
$\frac{\text{foot}}{\text{second}}$.

Whenever we make up a new kind of quantity by division of two other different kinds of quantity, the new quantity is called a **rate.**

It must not be thought that speed is the only example of a rate.

Here are others:—

(i)

						Unit.
Weight of water	187·5	250·0	125·0	437·5	62·5	*lb.*
Volume of water	3	4	2	7	1	*cubic ft.*
Rate	62·5	62·5	62·5	62·5	62·5	$\frac{lb.}{cubic\ ft}$

Here the unit of rate is a $\frac{\text{lb.}}{\text{cubic foot.}}$

or, better still, $\frac{\text{lb.}}{\text{foot}^3}$.

The actual rate above is 62·5 of these units.

The rate, $\frac{\text{lb.}}{\text{ft}^3}$, is used in Mechanics, where it is given the name 'density'

(ii)

						Unit.
Force exerted by air	15	45	60	75	105	*lb.*
Are exposed to air	1	3	4	5	7	*sq. in.* or in^2
Rate	15	15	15	15	15	$\frac{lb.}{in^2}$

Here the unit of rate is a $\frac{\text{lb.}}{\text{in}^2}$.

The actual rate above is 15 of these units.

Rate $= 15\frac{\text{lb.}}{\text{in}^2}$.

The rate $\frac{\text{lb.}}{\text{in}^2}$ is also used in Mechanics. It is called 'pressure.'

(iii)

					Unit.
Cost	$2\frac{1}{2}$	$7\frac{1}{2}$	10	$12\frac{1}{2}$	*shilling*
Number of Books	1	3	4	5	*book*
Rate	$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{2}$	$\frac{\textit{shilling}}{\textit{book}}$

Here the unit of rate is a $\frac{\text{shilling}}{\text{book}}$.

The rate is $2\frac{1}{2}$ $\frac{\text{shilling}}{\text{book}}$

or, as read, $2\frac{1}{2}$*s*. per book.

(iv)

					Unit.
Weekly Wages	50	150	200	300	*Rs*
Workmen	1	3	4	6	*man*
Rate	50	50	50	50	$\frac{\textit{Rs}}{\textit{man}}$

Here the unit of rate is a $\frac{\text{Rs}}{\text{man}}$.

The rate is 50 $\frac{\text{Rs}}{\text{man}}$.

or, as read, Rs 50 per man.

Example 1.

Express a speed of 50 miles an hour in feet per second.

$$\frac{50}{1}\ \frac{\text{mile}}{\text{hour}}$$

$$=\frac{50\times 5280}{1}\ \frac{\text{foot}}{\text{hour}}$$

$$=\frac{50\times 5280}{60\times 60}\ \frac{\text{foot}}{\text{sec.}}$$

$$=73\tfrac{1}{3}\ \frac{\text{foot}}{\text{sec.}}$$

or, as read, $73\frac{1}{3}$ feet per second. Ans.

Example 2.

Express a speed of 88 feet per sec. in miles per minute.

$$\frac{88}{1}\ \frac{\text{foot}}{\text{sec.}}$$

$$=\frac{88\div 5280}{1}\ \frac{\text{mile}}{\text{sec.}}$$ (to bring feet to miles divide by 5280).

$$=\frac{88\div 5280}{1\div 60}\ \frac{\text{mile}}{\text{min.}}$$ (to bring seconds to minutes divide by 60).

$$=\frac{88\times 60}{5280}\ \frac{\text{mile}}{\text{min.}}$$

$$=1\ \frac{\text{mile}}{\text{min.}}$$

or, 1 mile per minute. Ans.

Example 3.

Express a rate (density) of 7 lb. per cubic foot in ounces per cubic inch.

$$\frac{7}{1}\ \frac{\text{lb.}}{\text{ft.}^3}$$

$$=\frac{7\times 16}{1}\ \frac{\text{oz.}}{\text{ft.}^3}$$

$$=\frac{7\times16}{1728}\ \frac{\text{oz.}}{\text{in.}^3}$$

$$=\frac{7}{108}\ \frac{\text{oz.}}{\text{in.}^3}$$

$$=\frac{7}{108}\ \underline{\text{oz. per cubic inch.}}\ \text{Ans.}$$

Example 4.

Express a rate of 214½ yards in 13⅕ sec., in miles per hour.

$$\text{Rate }=\frac{214\frac{1}{2}}{13\frac{1}{5}}\qquad\frac{\text{yard}}{\text{sec.}}$$

$$=\frac{214\frac{1}{2}\div1760}{13\frac{1}{5}\div3600}\qquad\frac{\text{mile}}{\text{hour}}$$

$$=\frac{429}{2}\times\frac{5}{66}\times\frac{3600}{1760}\ \frac{\text{mile}}{\text{hour}}$$

$$=\underline{33{\cdot}2\text{ miles per hour.}}\ \text{Ans.}$$

Example 5.

Express 3·5 tons per 7·2 sq. ft. as cwt. per sq. yd.

$$\text{Rate }=\frac{3{\cdot}5}{7{\cdot}2}\qquad\frac{\text{ton}}{\text{sq. ft.}}$$

$$=\frac{3{\cdot}5\times20}{7{\cdot}2\div9}\qquad\frac{\text{cwt.}}{\text{sq. yd.}}$$

$$=\frac{35\times20\times9}{72}\qquad\frac{\text{cwt.}}{\text{sq. yd.}}$$

$$=\underline{87\tfrac{1}{2}\text{ cwt. per sq. yd.}}\ \text{Ans.}$$

Example 6.

A train travels 65 miles in 91 minutes. How many miles an hour is this ?

$$\text{Rate }=\frac{65}{91}\qquad\frac{\text{mile}}{\text{min.}}$$

$$=\frac{\frac{65}{91}}{1}\qquad\frac{\text{mile}}{\text{min.}}$$

(divide numerator and denominator by 91).

$$=\frac{\frac{65}{91}}{\frac{1}{60}} \qquad \frac{\text{mile}}{\text{hour}} \qquad \text{(reduce min. to hour by dividing by 60).}$$

$$=\frac{65}{91}\times\frac{60}{1} \qquad \frac{\text{mile}}{\text{hour}}$$

$$=\underline{42\tfrac{6}{7}\text{ miles per hour.}}\ \text{Ans.}$$

Example 7.

A motor car travels 77 yards in 4 sec. How many miles an hour is this ?

$$\text{Rate } =\frac{77}{4} \qquad \frac{\text{yard}}{\text{sec.}}$$

$$=\frac{\frac{77}{4}}{1} \qquad \frac{\text{yard}}{\text{sec.}}$$

$$=\frac{\frac{77}{4}\div 1760}{1\div 3600} \qquad \frac{\text{mile}}{\text{hour}} \qquad \text{(Reduce yards to miles and sec. to hours.)}$$

$$=\frac{77}{4}\times\frac{3600}{1}\times\frac{1}{1760} \qquad \frac{\text{mile}}{\text{hour}}$$

$$=\underline{39\tfrac{3}{8}\text{ miles per hour.}}\ \text{Ans.}$$

EXERCISES 28

Work out the following sums setting out your work clearly and methodically as has been done in the above examples. Name your units at each step.

Express

1. 30 yards per second in feet per minute.
2. 30 yards per second in feet per half minute.
3. 300 yards per second in miles per hour.
4. 44 feet per second in miles per hour.
5. 60 miles an hour in feet a second.
6. 40 miles an hour in feet a minute.
7. 20 miles an hour in furlongs a minute.
8. 560 miles a day in feet a second.
9. 70 lb. per sq. ft. in oz. per sq. in.
10. 140 lb. per sq. ft. in oz. per sq. yd.
11. 2 tons per sq. yd. in lb. per sq. ft.

12. 240 gm. per sq. metre in cg. per sq. cm.

13. 350 gm. per sq. metre in kg. per sq. cm.

14. 150 lb. per cubic foot in oz. per cubic inch.

15. Sound travels in air at the rate of 1100 ft per second. Express this velocity in miles per hour.

16. An aeroplane covered 4 miles in 1 min. 15 sec. What was its average speed in miles per hour ?

17. An aeroplane covered $5\frac{1}{2}$ miles in 1 min. 23 sec. Find its average speed in miles per hour (to the nearest $\frac{1}{10}$ mile per hour).

18. Police observers noticed that a car did the distance between two posts $1\frac{1}{2}$ miles apart in 103 seconds. At what rate was it travelling ?

19. Two cars do the distance between two places in 1 hour 20 min. and $1\frac{1}{2}$ hours respectively. Compare their speeds.

20. One motorist travels 5 miles in 4 seconds while another travels 4 miles in 5 seconds. Compare their speeds.

21. Which has the greater speed, a train which travels 800 yds in $1\frac{1}{2}$ minutes or a steamer which travels at 18 knots ? Express the difference in miles per hour. (1 knot is 'a nautical mile per hour'. 1 nautical mile=6081 feet.)

22. A time-keeper at a sports meeting stands at the winning post and judges the start of the 220 yards race by the report of the starter's gun. What correction should he make in the timing he obtains ?

23. A passenger in a train estimated that the distance between two cabins is covered in 24 seconds. If the distance between the cabins is 370 yards, find the speed of the train.

24. Express the following speeds (nearest $\frac{1}{10}$ unit) in miles per hour:

(*a*) 375 feet in 45 sec.

(*b*) 5500 yards in $2\frac{1}{3}$ min.

(*c*) $5\frac{1}{2}$ furlongs in 1 min. 23 sec.

(*d*) 5·6 furlongs in 1·6 min.

(*e*) 9 nautical miles in $12\frac{1}{2}$ min.

25. Express the following speeds in feet per sec. (nearest $\frac{1}{10}$ unit):

(*a*) 43 miles in $2\frac{1}{2}$ hours.

(*b*) 55 furlongs in 31 minutes.
(*c*) 887 yards in $\frac{1}{2}$ hour.
(*d*) 7 nautical miles in 13 minutes.
(*e*) 18 nautical miles in 29 minutes.

THE RATE USED AS A CONVERSION MULTIPLIER

Denoting time by t and distance by s,

$$\text{we have } t \times \frac{s}{t} = s,$$

$$\text{and } s \times \frac{t}{s} = t.$$

So, we can use the rate $\frac{t}{s}$ as a conversion multiplier to convert s to t; and the rate $\frac{s}{t}$ as a conversion multiplier to convert t to s.

Example 8.

A train runs $107\frac{1}{4}$ miles in $2\frac{3}{4}$ hours. How far does it travel in $3\frac{1}{4}$ hours?

$$\begin{aligned} \text{Distance} &= \text{hours} \times \frac{\text{miles}}{\text{hours}} \\ &= 3\tfrac{1}{4} \times \frac{107\frac{1}{4}}{2\frac{3}{4}} \text{ miles} \\ &= \frac{13}{4} \times \frac{429}{4} \times \frac{4}{11} \text{ miles} \\ &= \underline{\underline{126\tfrac{3}{4} \text{ miles.}}} \text{ Ans.} \end{aligned}$$

Example 9.

A train runs $107\frac{1}{4}$ miles in $2\frac{3}{4}$ hours. How long will it take to travel 143 miles?

$$\text{Time taken (hours)} = \text{miles} \times \frac{\text{hours}}{\text{miles}}$$

$$=143\times\frac{2\frac{3}{4}}{107\frac{1}{4}}$$

$$=143\times\frac{11}{4}\times\frac{4}{429}\text{ hours}$$

$$=\underline{3\tfrac{2}{3}\text{ hours.}}\text{ Ans.}$$

Example 10.

A car travels at the rate of 110 yards in 6 sec. How long will it take to cover a distance of 60 miles? (Answer in hours).

$$\text{Time}=\text{miles}\times\frac{\text{hours}}{\text{miles}}$$

$$=60\times\frac{6\div3600}{110\div1760}\text{ hours}$$

$$=60\times\frac{6}{110}\times\frac{1760}{3600}\text{ hours}$$

$$=\underline{1{\cdot}6\text{ hours.}}\text{ Ans.}$$

EXERCISES 29

1. A train travels 792 yards in 35 seconds. How long will it take to travel 550 yards?

2. A car was observed to travel the distance between two posts 500 feet apart in 11 seconds. How long did it take to cover the first 230 feet?

3. A body in uniform motion travels 37½ feet in 3¾ seconds. How long will it take to travel 25 feet?

4. A body in uniform motion travels 47·3 feet in 5 seconds. How far (nearest foot) will it travel in 13¾ seconds?

5. The distance between two stations, A and B, is 42·7 miles and the distance between B and C is 34·5 miles. A train does the distance from A to B in 1 hr 17 min. How long will it take to travel from B to C if it continues at the same rate? (Answer to the nearest minute).

6. A motorist travelled the distance between two towns 32½ miles apart in 1 hour 5 minutes. How long would he take to motor a distance of 67 miles if he drove uniformly at this rate?

7. An express train is observed to do the distance between two cabins 300 yards apart in 11 seconds. How long will it

take to do a journey of 85 miles if it travels uniformly at that rate? (Answer to the nearest minute).

8. Answer the following questions to the nearest $\frac{1}{10}$ of the unit specified:

(*a*) 71 miles are covered in 1 hr. 37 min.
1050 yds ,, in ? min.
(*b*) 83 furlongs ,, in 15 min. 35 sec.
? feet ,, in 35 min. 40 sec.
(*c*) 1900 yds ,, in 2 min. 20 sec.
? miles ,, in 2 hrs 40 min.
(*d*) 92 feet ,, in 1·6 sec.
? miles ,, in 5·5 hrs.
(*e*) 73 miles ,, in 2·7 hrs.
1400 yds ,, in ? min.
(*f*) 75 nautical miles ,, in 3·5 hrs.
10 (land) miles ,, in ? min.

9. A light engine is travelling at a speed which will enable it to reach a station $44\frac{1}{2}$ miles away in 56 minutes. How long will it take to pass a platform, en route, 94 yards long? (nearest second).

10. A train is booked to do the distance between two stations, 24 miles apart, in half-an-hour. If it travels at 45 miles an hour for the first quarter of an hour, at what speed must it run for the rest of the journey?

11. At the same time as one train leaves a station, A, for a station, B, another train leaves B for A. A and B are 42 miles apart and the trains pass each other at a spot 25 miles from A. Compare the speeds of the two trains.

12. Two motorists set out at the same time to motor to a place 80 miles away and back. The first goes at 40 m.p.h. and returns at 20 m.p.h. The second goes and returns at 30 m.p.h. Which motorist returns first?

13. A train is scheduled to do the distance between two stations 50 miles apart in an hour. If it travels the first 20 miles at 45 m.p.h., can it complete the journey in time without exceeding a speed restriction of 55 m.p.h.?

14. A ferry boat usually does the journey between two places in 1 hr 34 min. by maintaining an average speed of 23 m.p.h. By how much should it increase its speed in order to reduce the time taken by 10 minutes?

15. The distance between two 'ghats' A and B is $9\frac{1}{2}$ miles, and the distance between B and the 'ghat' C is $12\frac{3}{4}$ miles. A ferry boat leaves A for B at 8.30 a.m. and steams at 17 m.p.h. Another ferry leaves C for B at 8.40 a.m. At what speed should it steam in order to connect with the first ferry at B?

CHAPTER XIII

ADDITION AND SUBTRACTION OF RATES WORK: APPROACH SPEED

WORK

Consider the following examples:

Example 1.

One tap can fill a cistern in 3 hours. Another can fill it in 4 hours. How long will the cistern take to fill if both taps are opened together?

Here one tap fills at the rate of $\frac{1}{3}$ of the cistern per hour.

The other tap fills at the rate of $\frac{1}{4}$ of the cistern per hour.

Since the taps are opened together, the cistern is being filled at a rate equal to the sum of these rates.

The cistern is being filled at $(\frac{1}{3}+\frac{1}{4})$ of itself per hour.

The rate $=\frac{7}{12}$ in 1 hr.

$\therefore$ the whole cistern can be filled in $\frac{12}{7}$ hours

$=\underline{1\frac{5}{7}\text{ hours.}}$ Ans.

Example 2.

A supply tap can fill a cistern in 6 hours, while an outlet tap can empty it in 10 hours. If the tank is empty and both the taps are opened, how long will the cistern take to fill?

Here the respective rates are $\frac{1}{6}$ cistern per hour.
and $\frac{1}{10}$ cistern per hour.

The rate at which the cistern will fill is
$(\frac{1}{6}-\frac{1}{10})$ cistern per hour.
$=\frac{1}{15}$ per hour.

∴ the cistern will fill in 15 hours. Ans.

EXERCISES 30

1. Of two men, one can complete a piece of work in 5 hours and the other in 3 hours. How long will they take working together?

2. Three men working singly can do a piece of work in 7 days, 4 days and 5 days respectively. They set to work together to do it. How long will they take?

(Give your answer to the nearest $\frac{1}{2}$ day.)

3. Two taps, one of which can fill a cistern in 7 hours and the other in 10 hours, are opened together when the cistern is one third full. How long will they take to fill it?

(Answer to the nearest minute.)

4. Two taps can fill a cistern in 10 hours and 12 hours respectively and a third can empty it, if full, in 15 hours. If all three taps are opened together when the cistern is empty, how long will it take to fill?

5. Of two sums of money one can earn a certain amount of interest in three years; the other can earn the same interest in six years. How long will the combined sums take to earn this interest, the rate being the same in both cases?

6. Three men *A*, *B*, *C* can, if working alone, do a piece of work in 5 days, $7\frac{1}{2}$ days and 15 days respectively. If *A* and *B* are put on for two days and then *B* be taken off and *C* be put on to help *A*, how long will *A* and *C* take to complete the work?

7. *A* and *B*, working together can do a piece of work in 5 hours. *B* and *C* working together can do it in 8 hours, *C* and *A* working together in 4 hours. How long will each take working alone?

8. Two taps can fill a cistern in 12 hours and 15 hours respectively. If they are open together the supply of each diminishes by a quarter. How long will they take to fill the cistern if they are open together?

9. A cinema hall has two exits. If only one of them is open, the hall, full of people, can normally be emptied in 15 minutes. If the other alone is open, the hall can be emptied in 10 minutes. In how many minutes will it be possible to empty the hall if both exits are opened?

10. If in question (9) the second exit alone is opened for 5 minutes and then both opened, in how many minutes will it be possible to empty the hall?

11. Of two canvassers one brings in orders to the value of Rs. 500 every three weeks. The other brings in orders at the rate of Rs. 850 every fortnight. What value of orders per week is the firm receiving through these two canvassers?

(It is assumed that the orders come in at a uniform rate.)

12. A boy reckons that if he saves some of his pocket money at a certain rate per week, it will take him 12 weeks to save enough for an air gun. His friend reckons that at *his* rate of saving he will have to wait 20 weeks. They decide to pool their savings. In how many weeks will they be in a position to buy the gun?

What is the ratio of their contributions towards their purchase?

13. Five men or nine boys can do a piece of work in 12 days. How long will two men and three boys working together take?

14. Four men or seven boys can do a piece of work in 14 days. How long will seven men and four boys take?

(Answer to the nearest $\frac{1}{2}$ day.)

15. Six men and five boys can do a piece of work in 2 days. Four men and three boys take $3\frac{3}{4}$ days. How much faster does a man work than a boy?

SPEED

Addition and Subtraction

Example 3.

Two trains, one travelling at 60 miles an hour and the other at 45 miles an hour, leave two stations 70 miles apart at the same time and travel towards each other. When and where will they meet?

Since the trains travel towards each other, the rate at which the distance between them is being covered is equal to the *sum* of their rates.

The distance is being covered at (60+45) miles an hour.

105 miles in 1 hour.

70 in ?

Time taken $= \frac{70}{105}$ hr $= \frac{2}{3}$ hr $=40$ minutes.

The first train has now run $60 \times \frac{2}{3}$ miles
$=40$ miles.

The second train has run $45 \times \frac{2}{3}$ miles
$=30$ miles.

(Adding 40 and 30 we get 70: this checks the correctness of our answer.)

Answer: They meet after 40 minutes; 30 miles away from the station from which the slower train started.

Example 4.

A train which travels 50 miles an hour leaves a station and travels towards another station 60 miles away at the same time as a train which travels 40 miles an hour leaves the latter station for the former. When will the two trains be 45 miles apart?

The rate at which the distance between the trains is being covered is 90 miles per hour. When they are 45 miles apart, a distance of (60—45) or 15 miles has been covered.

They both cover 90 miles in 1 hour.

∴ they cover 15 miles in $\frac{15}{90}$ or $\frac{1}{6}$ hr
$=\underline{10 \text{ minutes}}$. Ans.

N.B.—There is another answer to this problem obtained by working with (60+45) miles instead of with (60−45) miles. Explain this.

Example 5.

A train which travels at 60 miles an hour leaves a station half an hour after, and travels in the same direction as another train travelling at 45 miles an hour has left the station. Where and when will the faster train overtake the other?

Since the trains are travelling in the same direction the distance between them is being diminished at a speed equal to the difference between their speeds.

The speed at which the distance is being diminished is therefore 15 m.p.h.

In $\frac{1}{2}$ hr the train that left first has gone $22\frac{1}{2}$ miles.

This distance is diminished at 15 miles in 1 hour and the second train will overtake the first in $\frac{22\frac{1}{2}\times 1}{15}$ hours, that is $1\frac{1}{2}$ hours.

$\therefore$ the faster train will overtake the other in $1\frac{1}{2}$ hours after it itself starts.

It will have travelled by then $60\times 1\frac{1}{2}$ miles $=90$ miles.

(The slower train has now been travelling 2 hours in all. It has, therefore gone 2×45 miles, that is 90 miles. This checks the answer.)

Example 6.

How long will two trains, of length 100 yards and 76 yards travelling in opposite directions at 40 and 45 miles an hour respectively, take to pass each other?

In working problems like this it is a good plan to keep one train fixed and let the other travel past it with the speed of approach. The speed of approach, or relative speed will be the sum of the two speeds if the trains are travelling in opposite directions. It will be equal to the difference of the two speeds, if the trains are travelling in the same direction.

Let us, in the present problem, keep the 100-yard train fixed; and let the other train travel past it at 85 miles an hour. Remembering that the engine of this train must travel the whole length of the other train plus the length of its own train before its train is clear of the other, we can state the problem thus:

To run 85×1760 yards it takes 3600 sec.

$\therefore$ To run 176 yards it will take

$\frac{3600\times 176}{85\times 1760}$ sec., that is $4\frac{4}{17}$ sec.

Example 7.

If in the last example, the trains were travelling in the same direction, our problem would be stated:

To run 5×1760 yards it takes 3600 sec.
To run 176 yards it will take
$\frac{176 \times 3600}{5 \times 1760}$ sec., that is 72 sec.

EXERCISES 31

1. A car left a place, *C*, and travelled at 30 miles an hour to a place, *D*, arriving there in 2 hrs 20 min. Where did it pass a car that left *D* for *C* at the same time as it left *C* and travelled at 20 miles an hour?

2. Two cars travelling at 40 and 25 miles an hour respectively leave two places at the same time and travel towards each other. The places are 91 miles apart. When and where will they meet?

3. Two cars travelling in opposite directions left two places $52\frac{1}{2}$ km apart at the same time. They passed each other after 42 minutes. It is known that one car travelled at 40 km an hour. What was the speed of the other car?

4. There are three stations *A*, *B*, *C*, on a railroad. *B* is 11 km from *A* and 15 km from *C*. At the same time as a train which travels at 44 km an hour leaves *A* for *C*, another train leaves *C* for *A*. How fast must the latter train travel if the two are to pass each other at B?

Which is the shortest way of working this problem?

5. A fast train which travels at 55 miles an hour leaves a station at the same time as a slow train, travelling in the same direction, leaves a station 93 miles away. The slow train travels at 15 miles an hour. How long after their departures will the first train overtake the slow train? How far out will they be?

6. Work question (5) assuming that the slow train got half an hour's start.

7. Work question (5) assuming that the fast train got half an hour's start.

8. A train leaves a station and travels at 32 km an hour. Three quarters of an hour later a fast train leaves the station

and travels in the same direction. At what speed must it travel so as to overtake the other train in $1\frac{1}{3}$ hours?

9. Two trains which travel at 40 and 25 miles an hour respectively leave a station, *A*, for a station, *B*, 55 miles away, at the same time as a train which travels at 30 miles an hour leaves *B* for *A*. How many minutes will elapse between the time when the first train passes the third and the time when the second train passes the third?

10. At the same time as one motorist leaves a place, *A*, and travels towards *B*, which is 32 miles away, another motorist leaves B. They travel in the same direction; the first at 33 miles an hour and the second at 21 miles an hour. The second motorist stops 20 minutes for refreshment at an inn after one hour's run and then resumes his journey. When and where will he be overtaken by the first?

11. A man can row a certain distance in still water in $1\frac{1}{2}$ hours. He can row the same distance down stream in $1\frac{1}{7}$ hours. How long will he take rowing upstream?

12. A boatman who can row at 8 km an hour in still water finds that he can row down the river from one *ghat* to another, 6 km distant, in 36 minutes. By how much must he increase his rate of rowing in order to row back in the same time ?

13. Two men walk, step by step, up an ascending escalator. One takes $\frac{3}{4}$ min. to get to the top and counts 55 steps. The other takes 1 min. and counts 30 steps. How long would they take to get to the top if they did not walk ? At how many steps to the minute is the escalator moving ?

14. A station-master heard the whistle of a train, and $1\frac{3}{4}$ min. later saw it steam into the platform. If the train whistled at a cabin 5 furlongs distant from the platform, what was its average speed between the cabin and the platform?

15. A gun is being fired at intervals of a second. At what intervals would the shots be heard by a motorist who is driving directly away from it at 25 m.p.h. ?

EXERCISES 32

1. How long will a train 75 yards long travelling at the rate of 40 miles an hour take to pass a platform 90 yards long?

2. If a train 80 yards long takes 8 sec. to pass a platform 96 yards long, how fast is it travelling?

3. A train 60 yards long travelling at 35 miles an hour takes 9 sec. to pass a platform. How long is the platform ?

4. A train travelling at 30 miles an hour passes a platform 80 yards long in 10 sec. How long is the train?

5. A train travelling at 30 miles an hour passes a platform 75 yards long in 10 sec. How long will it take to pass a platform 90 yards long, if it is travelling at the same rate ? (Answer to the nearest second.)

6. A train, 80 yards long, travelling at 40 miles an hour overtakes another, 95 yards long, travelling at 25 miles an hour. How long will it take to pass it completely?

7. A train, 65 yards long, travelling at 43 miles an hour overtakes another 70 yards long, travelling at 31 miles an hour. How long will it take to pass it completely?

8. Work Ex. 6, taking the two trains to be travelling in opposite directions. (Answer to the nearest $\frac{1}{10}$ sec.)

9. Work Ex. 7 taking the two trains to be travelling in opposite directions.

10. A train 77 yards long, travelling at 40 miles an hour, overtakes a man walking alongside the track in the same direction at 4 miles an hour. How long will it take to pass the man?

11. In question (10) how long would the train take to pass the man if it were travelling in the opposite direction to the man ? (Answer to the nearest $\frac{1}{5}$ sec.)

12. A train, 85 yards long, travelling at 42 m.p.h. overtakes another train travelling at 36 miles an hour, and passes it completely in $52\frac{1}{2}$ sec. How long is the second train?

13. A train travelling at 50 miles an hour overtakes a train, twice as long, travelling at 25 miles an hour, and passes it in $10\frac{4}{5}$ sec. What is the length of each train?

14. A train, 85 yards long, travelling at 45 m.p.h. in the opposite direction to another train travelling at 35 miles an hour, completely passes the latter in $4\frac{1}{2}$ seconds. How long would the former take if it were travelling only half as fast?

(i) How long would the trains take to pass each other if the former were travelling at half the speed, while the speed of the second was doubled?

(ii) How long would they take if the speed of the first train remained the same, while the speed of the second train halved?

(iii) How long would they take if the speeds of both trains were halved?

15. If a train, 70 yards long, travelled at 50 miles an hour, it would completely pass a train running in the opposite direction in $3\frac{15}{16}$ sec. If it travelled instead at 54 miles an hour it would pass the other in $3\frac{3}{4}$ sec. How long is the second train and how fast is it travelling? (Use algebraic symbols.)

CHAPTER XIV

PERCENTAGES

EXERCISES 33

1. Express as fractions 50%; 60%; 75%; $12\frac{1}{2}$%; $33\frac{1}{3}$%; 35%; x%.

2. Express as percentages

$\frac{2}{5}$; $\frac{3}{20}$; $\frac{1}{8}$; $\frac{1}{6}$; $\frac{3}{2}$; x/y.

3. Express the first quantity as a percentage of the second:

(*a*) 3*s*. 6*d*.; £1
(*b*) 7*s*. 6*d*.; £2
(*c*) 15*s*. 9*d*.; £8. 15*s*.
(*d*) £1.5*s*. 3*d*.; £101
(*e*) £13. 15*s*.; £35. 4*s*.

4. A man spends Rs. 304 a month. If he earns Rs. 320, what percentage of his income does he (*i*) spend (*ii*) save?

5. Find the difference between 8% of £ 320 and 13% of the same amount.

6. A bill amounts to £x. If 12% is deducted, the amount to be paid is £110; find the value of x.

7. A piece of cloth is measured, and the length is given as 12 metres when the correct length is 12·5 metres. Find the error per cent.

8. A piece of drill cloth is 50 cm long and 40 cm wide. When washed, its length shrinks by 10%, and its breadth by 5%. Find the percentage decrease in area.

9. A shop manager is paid a salary of Rs. 300 a month and receives a commission of $\frac{3}{4}$% on the sales. What were the sales in a month in which he received Rs. 780/- ?

10. Find the value after 3 years of a car bought for Rs. 8000, which loses each year 20% of its value at the beginning of that year.

11. In an election there were two candidates; one of them received 59% of the votes cast and so secured a majority of 54 votes. How many voted ?

12. An electric supply company raises its charges by 20% and a year later increases the new charges by 20%. If the total increase had been made all at once to what percentage would it have been equivalent?

13. The populations of two towns are in the ratio 5 : 7. If the population of the first goes up 26% while the other falls 25%, in what ratio will the population then be ?

14. The expenditure of an industrial concern is classified under three headings (*i*) salaries (*ii*) labour (*ii*) materials, and the expenses due to these three items are in the ratio of 3 : 4 : 8. If the salaries increased by 5%, the cost of labour by 12$\frac{1}{2}$% and the cost of materials by 10%, find the percentage increase in the total expenses of the industrial concern.

15. In July 1960 the export of a certain commodity was 4·7% greater in quantity and 22·8% greater in total value than in July 1959. What was the average percentage increase in the price of the commodity ? Give your answer correct to three significant figures.

16. There are three stones *X*, *Y* and *Z*. *X* is 15% heavier than *Y* and 35% heavier than *Z*. By how much per cent is *Z* lighter than *Y*? Give your answer correct to 2 figures.

17. The total income of a company for July 1960 was Rs. 325000 which after the payment of expenses left a balance of Rs. 12500/-. In August the total income increased by 10% and the expenses by 12%. Find the percentage change in the balance.

18. After the new budget in 1962 the price of petrol rose by 20%. By how much per cent must a motorist reduce his consumption of petrol so that his expenditure does not increase?

19. The duty on a certain commodity was reduced by 40% of its present amount. By how much per cent must the consumption increase so that (*i*) the revenue may be unaltered, (*ii*) the revenue may be increased by 10%?

20. My income was increased by 10% and later decreased by 10%; what is the total change per cent in my income?

CHAPTER XV

PERCENTAGE PROFIT AND LOSS
A BANKRUPT'S PAYMENTS

PERCENTAGE PROFIT AND LOSS

Suppose one man buys an article for Rs 20 and sells it for Rs 25, while another buys an article for Rs 50 and sells it for Rs 55. Both make a profit of Rs 5, but their rate of profit is different.

One makes Rs 5 on his outlay of Rs 20, while the other makes Rs 5 on an outlay of Rs 50. The state of affairs is more truly gauged by the rate $\frac{\text{Profit}}{\text{Cost Price}}$

In one case this rate is $\frac{5}{20}$ or $\frac{1}{4}$. In the other case it is $\frac{5}{50}$ or $\frac{1}{10}$.

Rates like $\frac{\text{Profit}}{\text{Cost price}}$ are peculiar because the constituent quantities are expressed in the same unit, namely, £ or Rs as the case might be. Nevertheless they are truly rates because we regard the two quantities (profit and cost price) as being different.

The rate $\frac{\textbf{Profit}}{\textbf{Cost Price}}$ is usually expressed as a percentage and then called the **percentage** profit.

In our example, one man makes 25% profit, the other man makes 10% profit.

Before dealing with problems in percentage profit and loss the different ways of 'translating' such phrases as 'a profit of 5%' or 'a loss of 12%' should be practised.

Thus 'a profit of 5%' may be analysed as

Profit	*Cost Price* C. P.	*Selling Price* S. P.
5	100	105

Always write 100 under C. P.

Now read: profit $= \frac{5}{100}$ of C. P.
profit $= \frac{5}{105}$ of S. P.
C. P. $= \frac{100}{5}$ of profit.
C. P. $= \frac{100}{105}$ of S. P.
S. P. $= \frac{105}{5}$ of profit.
S. P. $= \frac{105}{100}$ of C. P.

Example 1.

A man buys an article for a certain sum and sells it at a loss of 15%.

Write down the value of the following ratios:

(i) $\frac{\text{Loss}}{\text{Cost price}}$ (ii) $\frac{\text{Loss}}{\text{Selling price}}$ (iii) $\frac{\text{Cost price}}{\text{Selling price}}$

The required ratios are picked out from the single statement.

Cost price : *Loss* : *Selling price*
100 : 15 : 85

The required ratios are: (i) $\frac{15}{100}$ (ii) $\frac{15}{85}$ (iii) $\frac{100}{85}$.

Example 2.

Find the selling price of a car which is bought for Rs 307·50 and sold at a profit of 15%.

We first set out the statement

Cost Price : *Profit* : *Selling Price*
100 : 15 : 115

$$\text{The profit } = \frac{15}{100} \text{ of the cost price}$$

$$= \frac{15}{100} \text{ of Rs } 307{\cdot}50$$

=Rs 46·13 to the nearest nP.

The cost price =Rs 307·50.

The selling price =Rs 353·63. Ans.

Example 3.

Find the cost price of an article which is sold for Rs 467·40 at a profit of 23%.

First set out the statement

cost price	:	*profit*	:	*selling price*
100	:	23	:	123

The cost price $= \frac{100}{123}$ of the selling price

$= \frac{100}{123}$ of Rs 467·40

=Rs 380. Ans.

Example 4.

An article is sold for Rs 467·40 at a profit of 23%. For what should it be sold to make a profit of 30% ?

The ratio statement reads:

cost price	:	*profit*	:	*selling price*
100	:	23	:	123

The selling price $= \frac{123}{100}$ of the cost price.

But if a profit of 30% were to be made, the selling price would be $\frac{130}{100}$ of the cost price.

So the ratio of the new selling price to the former selling price is $\frac{130}{123}$.

∴ The new selling price $= \frac{130}{123}$ of Rs 467·40

= Rs 494. Ans.

Example 5.

An article was sold for Rs 467·40 at a profit of 23%.

What would have been the percentage gain or loss if it had been sold for Rs 399 ?

The question can be put as follows:

> Rs 467·40 represents 123% of the cost price. What percentage of the cost price would Rs 399 represent?

$$\text{The required percentage} = \frac{123}{467{\cdot}40} \times 399$$
$$= 105;$$

∴ there would have been a profit of 5%. Ans.

EXERCISES 34

1. Fill in the blanks below for 'translation' or equation of each of the following phrases:

'loss of 10%' ; 'gain of 10%' ; 'loss of 20%' ;
'profit of 21%' ; 'loss of 30%' ; 'gain of 29%' ;
(*i*) C.P. =. . . of S.P. (*ii*) S.P. =. . . of C.P.
(*iii*) Profit (or loss) =. . . of S.P. (*iv*) C.P. =. . . of profit (or loss).

2. Work out the alternative methods suggested in worked Examples 4 and 5 on pp. 97 and 98.

3. The following are to be worked mentally—only the answers to be written down:

(*a*) A profit of 10%; C.P. = Rs 210; S.P. = ?
(*b*) A profit of 10%; S. P. = Rs 132; C.P. = ?
(*c*) A profit of 15%; C.P. = Rs 240; S.P. = ?
(*d*) A loss of 15% ; C.P. = Rs 240; S.P. = ?
(*e*) A loss of 15% ; S. P. = Rs 170; C.P. = ?
(*f*) A gain of 23% ; S. P. = 369 frs.; C.P. = ?
(*g*) A loss of 23% ; Loss = £69 $\left\{\begin{array}{l}\text{C.P.} = ?\\ \text{S.P.} = ?\end{array}\right.$
(*h*) A gain of 33%; Gain = £132; S.P. = ?

EXERCISES 35

1. Find the actual profit or loss and the selling price when an article—

(*a*) costing Rs 350·50 is sold at a profit of 8%.
(*b*) costing Rs 730·80 is sold at a profit of $8\frac{1}{3}$%.
(*c*) costing Rs 5200 is sold at a loss of 21%.
(*d*) costing £ 5333 6*s*. 8*d*. is sold at a loss of $6\frac{3}{4}$%.

2. What is the percentage profit or loss if an article—
   (*a*) costing Rs 6637 is sold for Rs 7367·07 ?
   (*b*) costing Rs 350·40 is sold for Rs 402·96 ?
   (*c*) costing Rs 358 is sold for Rs 370·53 ?
   (*d*) costing £ 525 is sold for £ 481 19*s*. ?

3. What is the cost price of an article which is sold for—
   (*a*) Rs 77·07 at a profit of 5% ?
   (*b*) Rs 773·76 at a loss of 7% ?
   (*c*) Rs 1530 at a loss of 12½% ?
   (*d*) $695 53c. at a loss of 8⅓% ?

4. Find the cost price of a horse that is being sold for Rs 1200 at a profit of 20%.

5. Find the cost price of an article which, if sold for Rs 103·50 would realise a profit of 15%.

6. A man finds that, if he sells an article for Rs 51, he will lose 15%. What did the article cost him ?

7. A man makes a profit of 15% by selling an article for Rs 250. What percentage profit would he make if he sold it for Rs 300 ?

8. A man finds that by selling an article for 312 francs he makes a profit of 4%. For what should he sell it to make a profit of 8% ?

9. If a profit of 8⅓% were desired, an article would have to be sold at a profit of 259 fr. 20 cents, for what would it have to be sold in order to make a profit of 12½% ?

10. An article was sold for Rs 861 at a gain of 23% on the cost price. What would have been the percentage gain had it been sold for Rs 798 ?

11. There would be a gain of 17% if an article were sold for £397 12*s*. What would be the percentage gain or loss if it were sold for £323 ?

12. There would be a loss of 5¾% if an article were sold for £1753 1*s*. What would be the percentage gain or loss if it were sold for £1966 19*s*. ?

13. *A* buys an article for Rs 65 and sells it to *B* at a profit of 20%. *B* sells it to *C* at a loss of 20%. What does *C* pay for it ?

14. *A* buys an article for $32 50 cents and sells it *B* to at

a profit of 20%. *B* sells it to *C* at a profit of 20%. What does *C* pay for it ?

15. *A* buys an article and sells it to *B* at a profit of 15%. *B* sells it back to *A* at the price which *A* previously paid for it. What is *B*'s percentage loss ?

EXERCISES 36

1. A man buys eggs at 1*s*. 8*d*. a doz. and sells them at 3*s*. 4*d*. a score. What is his percentage profit ?

2. A man buys 750 oranges at the rate of Rs 13·50 a hundred. At what price per dozen should he retail them so as to make a profit of Rs 48·75. What percentage profit does this represent ?

3. Pencils which cost Rs 37·50 a thousand to produce are sold for Re. 1 a dozen. What is the percentage profit ?

4. What is the gain per cent if kites are bought at 9 for 50 nP. and sold at the rate of 8 for 60 nP. ?

5. A man marks the selling price of his goods on the basis of a 20% profit on his cost price. He, however, allows a discount of 10% off the marked price to customers who pay cash. What was the dealer's cost price of an article for which a customer paid Rs 6·75 cash ?

6. A dealer buys 31 doz. oranges at 10*d*. a doz. and 16 doz. apples at 1*s*. 3*d*. a doz. He sells the oranges at a profit of 10% and the apples at a profit of 21%. What percentage profit does he make on the whole ?

7. A fruiterer has 500 oranges which he bought at the rate of Rs 2 a score. He has also 480 apples which cost him Rs 1·50 a doz. He has to sell the oranges out at a loss of 10%. What per cent profit must he make on the apples so that he will be square on the joint transaction ?

8. A sports dealer has to sell out a consignment of 120 tennis racquets which cost him 25*s*. each. He finds that he can sell them, if properly advertised, at 35*s*. each. If he is content with a profit of 15%, what amount can he afford to spare on the advertisement and other charges ?

9. If a milkman dilutes 40 gallons of milk with 5 gallons of water, but does not alter his usual price, what percentage profit calculated on his usual price does he make ?

10. A cutler finds that of his stock of 19 doz. knives

2 doz. are shopsoiled. The latter he has to sell out at a reduction of 30%. The knives cost him Rs 10·50 a dozen. His usual selling price is Rs 2 each. What is the percentage profit on the entire stock ?

11. A toy-shopkeeper finds that out of a lot of 75 toys 18 are damaged and unsalable. The toys cost him 3*s*. 6*d*. each. His selling price is 4*s*. 8*d*. What is his percentage profit ?

12. A dealer finds that if he has to pay £5 more for an article he will have to enhance his selling price by £9 in order to make the same percentage profit as before. What percentage profit is he making ?

BANKRUPTCY

A **debtor** is a man who owes money. A **creditor** is a man to whom money is owed.

When a debtor cannot pay all his debts he is said to be **insolvent.** He and his creditors may come to some agreement between them as to the terms of settlement of the debt; or, either party, under certain conditions, may have recourse to the **Bankruptcy** Court.

If the Court after investigation is satisfied that the debtor is insolvent it declares him a **bankrupt.*** It takes over charge of his affairs. All the money he has and all his disposable property (within certain legal restrictions) are used towards the discharge of his debts. The total amount available for payment of his debts is called his **assets.** What he actually owes are his **liabilities.**

Suppose a man's assets are Rs. 6000, while his liabilities amount to Rs. 8000. He will be able to pay his creditors only $\frac{6}{8}$ i.e. $\frac{3}{4}$ of what he owes, and his creditors are required to accept this fraction of what is owed to them in full discharge of the debt.

The rate at which a bankrupt discharges his debts is called the **composition.** The composition is expressed as so many nP. on the Re or so many shillings on the £, as the case may be. In our example the composition would be expressed as 75 nP. on the Re.

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* In England a bankrupt is an insolvent person judged to be so by the Court. In India it has become the custom to use 'insolvent' to cover both meanings.

Conversely, if the composition is 48 nP. on the Re we may infer that the debtor's

$$\frac{\text{assets}}{\text{liabilities}} = \frac{48}{100} = \frac{12}{25}.$$

So his assets $= \frac{12}{25}$ of his liabilities
or his liabilities $= \frac{25}{12}$ of his assets.

Example 6.

A man owes three creditors sums of Rs 7000, Rs 3200 and Rs 4800. His assets, however, are only Rs 5000. At what rate will he be able to pay his creditors and what will he actually pay each of them ?

His total liabilities are Rs 7000+3200+4800, that is, Rs 15000.

He will be able to pay Re $\frac{5000}{15000}$ on each rupee, i. e. Re $\frac{1}{3}$ on each rupee.

Each creditor will accordingly receive one-third of what is owed to him.

The 1st will receive $\frac{1}{3}$ of Rs 7000 that is Rs 2333·33
The 2nd will receive $\frac{1}{3}$ of Rs 3200 that is Rs 1066·67
The 3rd will receive $\frac{1}{3}$ of Rs 4800 that is Rs 1600·00
Total payments (=assets) Rs 5000·00

EXERCISES 37

1. A bankrupt's assets are Rs 7500 and his liabilities to two creditors Rs 5000 and Rs 4000. At what rate can he pay and what will each creditor receive ?

2. A bankrupt is able to pay two-thirds of his debts. What are his liabilities if his total assets amount to Rs 3500 ?

3. A bankrupt is able to pay 90 nP. on the rupee. What are his liabilities if his total assets amount to Rs 7100 ?

4. A bankrupt owes Rs 3625 and his liabilities amount to Rs 5000. What is he able to pay on the Re after meeting court dues at the rate of 10% of the liabilities ?

5. A bankrupt's assets are 65% of the liabilities. What can he pay on the £ ? What will a creditor receive to whom he owes £321 15*s*. ?

6. How much does a creditor lose on payment from a bankrupt who owes him £3215 15*s*., but can pay only 13*s*. 6*d*. on the £ ?

7. What percentage of his money does a creditor receive who is paid at the rate of 50 nP. on the rupee by one bankrupt who owes him Rs 7000, and at the rate of 75 nP. on the rupee by another bankrupt who owes him Rs 5000 ?

8. What percentage of his dues does a creditor receive who is paid at the rate of 50 nP. on the rupee by one debtor who pays him Rs 7000, and at the rate of 75 nP. on the rupee by another debtor who pays him Rs 5000 ?

CHAPTER XVI

SIMPLE INTEREST

Interest is money paid for the use of money borrowed. It is quoted as a percentage of the money borrowed. The money borrowed or lent is called the **principal.** Thus if £450 is borrowed at 5% interest per annum, the borrower will have to pay the lender 5% of £450 as interest at the end of every year.

Banks pay interest to people who 'deposit' money with them. People do not put (deposit) their savings in a bank merely because it is inconvenient and unsafe to keep it with themselves, but because there is the further inducement that the bank will pay them interest. The bank really borrows the money from them. (The money deposited with the bank does not lie idle there.) The bank in its turn lends the money (at a greater rate of interest) to business people to enable them to carry on their business, and it is thus able to pay interest to people (who deposit money with it).

When a man deposits his savings at a bank, the bank gives him no special surety that his money is safe. The depositor relies on the good name and soundness of the bank. If the bank fails, the man stands to lose some or all of his money.

When the bank lends money it usually requires a surety or 'security'. If the borrower fails to pay back his loan the bank is safeguarded, as the borrower then forfeits the security.

An important form of security is a building or estate. Landlords and companies often borrow money on their houses or factories as security. This loan enables them to extend and carry on their business. They are charged interest on the loan, and, if they fail to pay back the loan within the stipulated period, they forfeit the building or estate which they gave as 'security'.

When a building or estate is made over as security on a loan it is said to be **mortgaged.** The loan is said to be raised on a mortgage.

When the interest is paid to the lender regularly every year or every half-year, we call the interest **Simple Interest.**

When money is lent or borrowed at **Compound Interest** the interest is not paid year by year, but is added to the principal at the end of every year; so that the principal for each succeeding year is the amount at the the end of the previous year.

SIMPLE INTEREST

Calculation of Simple Interest

Study the following:

1. Simple Interest on Rs 200 for 1 year at 4% is Rs 8.
   The amount at the end of the year is Rs 208.
2. Simple Interest on Rs 200 for 2 years at 4% is Rs 16.
   The amount at the end of the two years is Rs 216.
3. Simple Interest on Rs 200 for 3 years at 4% is Rs 24.
   The amount at the end of the three years is Rs 224.

From the above examples it is clear that simple interest is proportional to the time for a fixed principal and a fixed rate; but the amount is NOT proportional to the time.

Example 1.

Method I:

Find the simple interest on Rs 350 for 5 years at $2\frac{1}{2}$%. Find also the amount.

Simple Interest on Rs 100 for 1 year is Rs $2\frac{1}{2}$.

$\therefore$ Simple Interest on Rs 100 for 5 years is Rs $2\frac{1}{2}\times 5=$ Rs $12\frac{1}{2}$.

$\therefore$ Simple Interest on Rs 350 for 5 years is $\dfrac{\cancel{25}\times\overset{175}{\cancel{350}}}{\cancel{2}\times\underset{4}{\cancel{100}}}$

$$= \text{Rs } \frac{175}{4}.$$

$$\left.\begin{array}{r} = \text{Rs } 43{\cdot}75. \\ \therefore \text{ Amount} = \text{Rs } 350 + \text{Rs } 43{\cdot}75 = \underline{\underline{\text{Rs } 393{\cdot}75.}} \end{array}\right\} \text{Ans.}$$

Method II:

The method in Example 1 must first be worked. This method may be used to obtain a general formula.

Find the simple interest on Rs P for Y years at R per cent per annum.

Simple Interest on Rs 100 for 1 year is Rs R.

$\therefore$ Simple Interest on Rs 100 for Y years is Rs R × Y.

$\therefore$ Simple Interest on Rs P for Y years is Rs $\dfrac{R\times Y}{100}\times P$.

$\therefore$ Simple Interest = Rs $\dfrac{P\,R\,Y}{100}$.

Hence the formula:

$$I = \frac{P\,R\,Y}{100} \text{ where}$$

P is the *Principal* in Rupees.
R is the *rate* per cent per annum.
Y is the *number of years*.
I is the *simple interest*.

$$= \frac{350\times 2\frac{1}{2}\times 5}{100}$$

$$\left.\begin{array}{r} = \dfrac{175}{4} = \underline{\underline{\text{Rs } 43{\cdot}75.}} \\ \therefore \text{ Amount } = \underline{\underline{\text{Rs } 393{\cdot}75.}} \end{array}\right\} \text{Ans.}$$

Example 2.

Find the Simple Interest on £ 251 13*s*. 4*d*. for 18 months at $4\frac{1}{2}$ per cent per annum.

$$I = \frac{P\,R\,Y}{100}.$$

P = £ $251\frac{2}{3}$; R = $4\frac{1}{2}$; Y = 18 months = 3/2 years.

$$\text{Interest} = \frac{\overset{151}{\cancel{755}}}{\cancel{3}} \times \frac{9}{2} \times \frac{\cancel{3}}{2} \times \frac{1}{\underset{20}{1\cancel{0}\cancel{0}}} = \frac{\pounds\ 1359}{80}$$

$$= \underline{\pounds\ 16\ 19s.\ 9d.}\ \text{Ans.}$$

N.B. In the formula Y is the number of years. We must express the 18 months in years i.e. 3/2 years.

Example 3.

Find the Simple Interest on Rs 324·46 for 2½ years at 3½% per annum. Give your answer to the nearest nP.

$$I = \frac{P\,R\,Y}{100}.$$

$P = \text{Rs } 324{\cdot}46;\ R = 3\tfrac{1}{2};\ Y = 2\tfrac{1}{2}.$

$$I = \frac{324{\cdot}46 \times 7 \times 5}{100 \times 2 \times 2} = \frac{11356{\cdot}1}{4 \times 100}$$

$$= \text{Rs } \frac{2839{\cdot}025}{100}$$

$$= \underline{\text{Rs } 28{\cdot}39}\ \text{to the nearest nP. Ans.}$$

EXERCISES 38

1. Find the Simple Interest on Rs 350 for 7 years at 3½% per annum.

2. Find the Simple Interest on Rs 278 for 4 years at 4¼% per annum.

3. Find the Simple Interest on Rs 87·60 for 5 years at 3% per annum.

4. Find the Simple Interest on Rs 1250 for 6 years at 3% per annum.

5. Find the Simple Interest on £ 6550 for 1½ years at 4% per annum.

Find the Simple Interest on, and the amount, at Simple Interest of:

6. Rs 1840 for 2 years at 4½%.
7. Rs 728 for 18 months at 2½%.
8. Rs 375 for 33 months at 2⅔%.

9. £ 187 10*s*. for 27 months at $3\frac{1}{5}\%$.

10. £8675 for 16 years at $2\frac{1}{8}\%$.

Example 4.

At what rate will the Simple Interest on Rs 7560 amount to Rs 1134 in 5 years?

$$I = \frac{P\ R\ Y}{100};$$

$$\therefore \quad 1134 = \frac{\overset{378}{\cancel{7560}} \times R \times \cancel{5}}{\underset{20}{\cancel{100}}}$$

$$\text{or} \quad 378R = 1134$$

$$\text{or} \quad R = \frac{1134}{378} = 3.$$

3% Ans.

Example 5.

In what time will the Simple Interest on Rs 7560 be Rs 1134 at 3% per annum?

$$I = \frac{P\ R\ Y}{100}$$

$$\text{or} \quad 1134 = \frac{\overset{378}{\cancel{7560}} \times 3 \times Y}{\underset{5}{\cancel{100}}}$$

$$\text{or} \quad 1134 = \frac{378 \times 3 \times Y}{5}$$

$$\therefore \quad Y = \frac{\overset{3}{\cancel{1134}} \times 5}{\cancel{378} \times \cancel{3}}$$

$$= \underline{5 \text{ years.}} \text{ Ans.}$$

Example 6.

What principal will yield Rs 567 at 3% per annum simple interest in 5 years?

$$I=\frac{P\ R\ Y}{100}$$

$$\text{or } 567=\frac{P \times 3 \times 5}{100}$$

$$\text{or } P=\frac{\overset{189}{\cancel{567}} \times \overset{20}{\cancel{100}}}{\cancel{3} \times \cancel{5}}$$

$$=3780.$$

Rs 3780. Ans.

Example 7.

Find the simple interest on Rs 785 at 4% per annum from May 12 to July 24.

Number of days	May	19*
	June	30
	July	24
		73

$$I=\frac{P\ R\ Y}{100}$$

$$=\frac{\overset{157}{\cancel{785}} \times 4 \times \cancel{73}}{100 \times \underset{5}{\cancel{365}}}$$

$$=\frac{628}{100}=6{\cdot}28.$$

Rs 6·28 Ans.

***Note**

When calculating the interest for a period between two given dates assume that the year contains 365 days. Omit the day the money was deposited in the bank, but COUNT the day when it is withdrawn as shown in example 7.

Example 8.

Find the Simple Interest on £ 272 4*s*. 7*d*. at $3\frac{1}{2}\%$ for $2\frac{1}{4}$ years. Give the answer correct to the nearest penny.

As the principal contains an awkward fraction of a £ it is better to use decimals. Work to four places of decimals.

4*s*. 7*d*. = £ 0·2292.

$$\therefore \mathrm{I} = \frac{\mathrm{P\ R\ Y}}{100} = \frac{272{\cdot}2292 \times 7 \times 9}{100 \times 2 \times 4}$$

$$= \frac{1905{\cdot}6044 \times 9}{800} = \frac{17150{\cdot}4396}{800} = 21{\cdot}4380$$

$$= \underline{\pounds\ 21\ 8s.\ 9d.}\ \text{Ans.}$$

Example 9.

Find the principal which amounts to Rs 654 in 3 years at 3% simple interest.

Simple Interest on Rs 100 at 3% for 3 years = Rs 9.

∴ Amount is Rs 100 + Rs 9 = Rs 109.

When the amount is Rs 109 the principal is Rs 100.

∴ When the amount is Rs 654 the principal is $\text{Rs}\,\dfrac{100 \times \overset{6}{\cancel{654}}}{\cancel{109}}$

$$= \underline{\text{Rs } 600.}\ \text{Ans.}$$

EXERCISES 39

1. Find the Simple Interest on
   - (*a*) Rs 7200 for 3 years at 6% per annum.
   - (*b*) £ 9560 for 2 years at 3% per annum.
   - (*c*) £ 9560 for 3 years at 2% per annum.
   - (*d*) Rs 1225 for $4\frac{1}{2}$ years at $3\frac{1}{3}$% per annum.
   - (*e*) £ 112 10*s*. for 4 years at $1\frac{1}{2}$% per annum.

2. Find at what rate the principal of
   - (*a*) £ 450 will yield an interest of £ 135 in 5 years.
   - (*b*) Rs 7325 will yield an interest of Rs 586 in 4 years.
   - (*c*) £ 640 will yield an interest of £ 12 in 3 months.
   - (*d*) Rs 2000 will yield an interest of Rs 112·50 in 15 months.
   - (*e*) Rs 12880 will yield an interest of Rs 2213·75 in 6 years 3 months.

3. In what time will
   (*a*) £ 5620 amount to £ 5957 4*s*. at 3 % per annum?
   (*b*) £ 285 amount to £ 364 16*s*. at 8 % per annum?
   (*c*) Rs 7850 amount to Rs 8635 at 4 % per annum?
   (*d*) £ 1830 amount to £ 2035 17*s*. 6*d*. at 4½ % per annum?
   (*e*) £ 6318 15*s*. amount to £ 7645 13*s*. 9*d*. at 4 % per annum?

4. Find the principal that will yield an interest of
   (*a*) Rs 81 in 9 years at 4 % per annum.
   (*b*) Rs 124 in 2½ years at 8 % per annum.
   (*c*) £ 1316 14*s*. in 6 years at 3 % per annum.
   (*d*) Rs 367·5 in 4½ years at 3⅓ % per annum.
   (*e*) £ 72 14*s*. 9*d*. in 4 years at 5½ % per annum.

5. (*a*) At what rate will £6250 amount to £7000 in 4 years?
   (*b*) At what rate will Rs 3575 amount to Rs 3932·50 in 2 years?
   (*c*) In what time will £8695 amount to £9999 5*s*. at 3 %?
   (*d*) In what time will Rs 4448 amount to Rs 4670·40 at 2½ %?
   (*e*) In what time will 73865 fr. amount to 79035 fr. 55 cents at 3½ %?
   (*f*) At what rate will 65375 fr. amount to 78450 fr. in 5 years?

6. (*a*) Find the sum which will amount to £ 165 15*s*. at 3 % per annum in 3½ years simple interest.
   (*b*) Find the principal which will amount to Rs 364·80 at 3½ % per annum in 8 years at simple interest.
   (*c*) Find what sum will amount to £140 17*s*. 6*d*. in 4½ years at 3⅓ % per annum simple interest.
   (*d*) Find what sum will amount to £227 5*s*. 9*d*. in 17 years at 3½ % per annum simple interest.
   (*e*) What sum of money will amount to £744 in 4 years at 5 % simple interest?

EXERCISES 40

1. Which receives more interest: £60 invested at 4 per cent or £95 at 2½ per cent? What is the annual difference?

2. A man has a bill for Rs 65. Simple interest is charged as long as the bill is not cleared. If the rate of interest is 6%, how much will he have to pay when he clears it after it is 18 months overdue?

3. How long will it take Rs 100 to double itself at 5% simple interest? What will be the length of time if it is £ x?

4. At the end of 24 years a sum of money is doubled; what is the rate of interest?

5. A man had Rs 7000 to invest. He invests Rs 4000 at 3%. At what interest must he invest the remainder in order to obtain a 4% interest on the total sum?

6. What is the interest per £ per month if the Post Office offers $2\frac{1}{2}$% per annum?

7. A man deposits £ 1260 in a fixed deposit for 5 years at simple interest. What amount will stand to his credit after 5 years if the rate is $3\frac{3}{4}$% per annum.

8. The interest on a certain sum of money deposited for 6 years at 3% simple interest is £306. What would the interest be were it deposited for 2 years at 5%?

9. Prove that the Simple Interest on any sum of money at $7\frac{1}{2}$% is 0·075 of the principal.

10. The yearly interest on a certain sum of money is Rs 5·50 greater if it is deposited at 3 per cent than if it is deposited at $2\frac{1}{2}$ per cent. What is the sum?

11. By how much is the yearly interest on Rs 666 increased if it is deposited at $4\frac{3}{4}$ per cent instead of at $3\frac{1}{2}$ per cent S. I.?

12. A sum of money deposited for 5 years at $3\frac{1}{2}$ per cent earns Rs 103·20 less interest than if it were deposited at $4\frac{1}{4}$ per cent for the same time. What is the sum?

13. A sum of money loaned for 6 years at $2\frac{1}{2}$ per cent earns Rs 710·50 more interest than if it were deposited for 9 years at $1\frac{1}{3}$ per cent. What is the sum?

14. Rs 7000 left in a bank for 10 years at simple interest amounts to Rs 9800. What would it amount to after another five years at the same rate?

15. A sum of money left in a bank at $3\frac{1}{2}$ per cent S. I. amounted to £10125 in 10 years. What would it have amounted to if it had been left for only 9 years?

CHAPTER XVII

LOGARITHMS

If a number is expressed as a power of 10 the INDEX is called the common LOGARITHM of the number.

$$\sqrt[4]{10^1}=10^{\frac{1}{4}}=10^{\frac{2}{8}}=1{\cdot}778=1{\cdot}78.$$

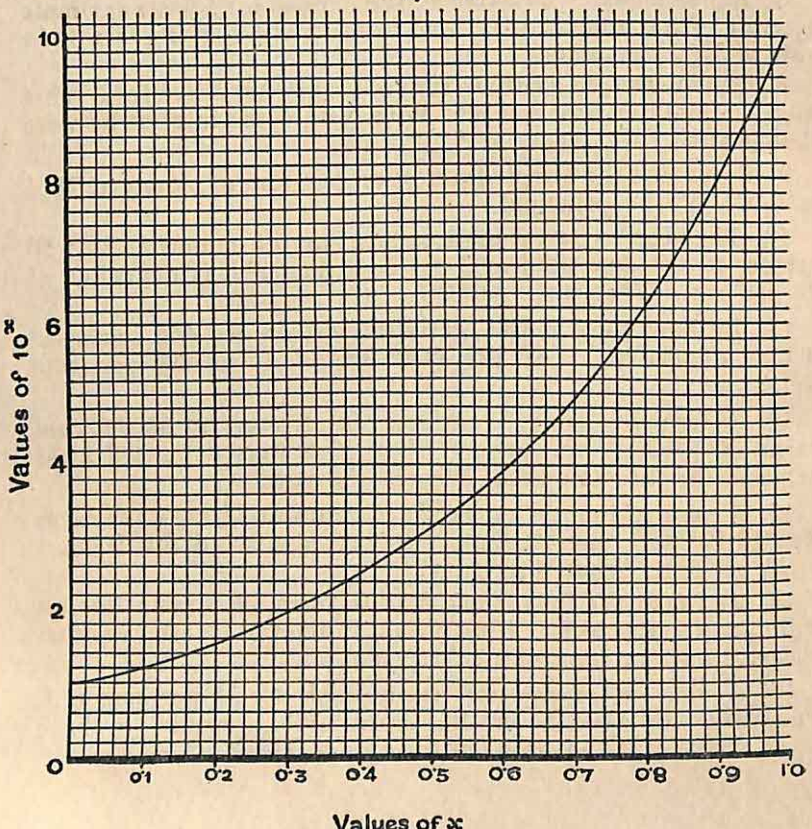
$$\sqrt[4]{10^2}=10^{\frac{2}{4}}=10^{\frac{4}{8}}=3{\cdot}162=3{\cdot}16.$$

$$\sqrt[4]{10^3}=10^{\frac{3}{4}}=10^{\frac{6}{8}}=5{\cdot}623=5{\cdot}62.$$

Table for the graph of 10^x:

x	0	$\frac{1}{8}=0{\cdot}125$	$\frac{1}{4}=0{\cdot}25$	$\frac{3}{8}=0{\cdot}375$	$\frac{1}{2}=0{\cdot}5$	0·625	0·75	0·875	1
10^x	1	1·33	1·78	2·37	3·16	4·22	5·62	7·50	10

Graph of 10^x



(1) Use the graph of 10^x and write down the values of $10^{0.2}$; $10^{0.7}$; $10^{0.4}$; $10^{0.6}$; $10^{0.34}$.

(2) Express as powers of 10

2; 3; 4; 7; 8·5; 2·5; 1·5.

Note

$10^a \times 10^b = 10^{a+b}$.

$10^a \div 10^b = 10^{a-b}$.

From the graph, 3=0·48 (approx).

∴ The logarithm of 3 is 0·48

Four-figure logarithm Table for

Numbers between 1 and 10

*	0	1	2	3	...	9	1	2	3	7	8	9
3·0	·4771	4786	4800	4814	...	4900	1	3	4	10	11	13
3·1	4914	4928	4942	4955	...	5038	1	3	4	10	11	12

3 $= 10^{\cdot 4771}$; 3·01 $= 10^{\cdot 4786}$; 3·09 $= 10^{\cdot 4900}$

3·1 $= 10^{\cdot 4914}$; 3·13 $= 10^{\cdot 4955}$ = 3·19 $= 10^{\cdot 5038}$

3·013 $= 10^{\cdot 4790}$:

Number	Logarithm
3·01	·4786
Diff. for 3	4
3·013	·4790

* To save space the decimal point in the left-hand column and in the main columns are usually omitted and must be supplied by the reader.

Now $10^0 = 1$ and $10^1 = 10$; therefore the logarithm of any number between 1 and 10 lies between 0 and 1 and so is written as 0·****, and the figures printed in the (four-figure) logarithm tables are the figures of the logarithm which follow the decimal point though the decimal point is not printed.

EXERCISES (Oral) 41A

Use tables to express the following as powers of 10:

1. 2·2.
2. 3·4.
3. 5·6.
4. 9·7.
5. 3.
6. 5.
7. 8.
8. 2·4.
9. 4·1.
10. 5·3.
11. 2·12.
12. 3·23.
13. 4·26.
14. 5·37.
15. 4·01.
16. 5·05.
17. 5·09.
18. 7·18.
19. 7·23.
20. 8·99.
21. 4·216.
22. 3·217.
23. 8·123.
24. 8·596.
25. 7·326.
26. 6·119.
27. 6·109.
28. 7·209.
29. 9·909.
30. 8·887.
31. 1·001.
32. 3·002.
33. 2·005.
34. 6·050.
35. 8·081.

Example 1.

Find the value of

(*i*) $10^{\cdot 3802}$ (*ii*) $10^{\cdot 4928}$ (*iii*) $10^{\cdot 4938}$.

(*i*) Look for the figures ·3802 in the 4-figure columns; these are found in the "24" row in the column headed 0. Therefore the significant figures in the value of $10^{\cdot 3802}$ are 240. Now the value of $10^{\cdot 3802}$ lies between 10^0 and 10^1 that is between 1 and 10.

$\therefore 10^{\cdot 3802} = \underline{2{\cdot}400 \text{ to 4 figures}}$. Ans.

(*ii*) The figures ·4928 are found in the "31" row in the column headed 1.

$\therefore 10^{\cdot 4928} = \underline{3{\cdot}110 \text{ to 4 figures}}$. Ans.

(*iii*) The actual figures 4938 do not occur in the four-figure columns.

$$10^{\cdot 4928} = 3{\cdot}110 \quad \text{and} \quad 10^{\cdot 4942} = 3{\cdot}120.$$

∴ The value of $10^{\cdot 4938}$ lies between 3·110 and 3·120

The difference between 4928 and 4938 is 10 and in the column of mean differences for this row 10 occurs under the column headed 7.

$\therefore 10^{\cdot 4938} = \underline{3{\cdot}117 \text{ to 4 figures}}$. Ans.

EXERCISES (Oral) 41B

1. Use logarithm tables to find the values of

$10^{\cdot 7404}$;
$10^{\cdot 9566}$;
$10^{\cdot 781}$;
$10^{\cdot 7435}$;
$10^{\cdot 9581}$;
$10^{\cdot 8258}$;
$10^{\cdot 7456}$;
$10^{\cdot 6293}$;
$10^{\cdot 5286}$;
$10^{\cdot 9138}$;
$10^{\cdot 8642}$;
$10^{\cdot 3593}$;
$10^{\cdot 9185}$.

2. Use logarithm tables to find the numbers of which the logarithms are

0·7962; 0·8319; 0·2532; 0·7856;
0·9701; ·0298; 0·0029; 0·0092;
0·6596; 0·7359; 0·9997; 0·0004.

Multiplication and Division:

Example 2.

Multiply 3·07 by 2·15. Rough Answer is 6.

$3{\cdot}07 \times 2{\cdot}15 = 10^{0\cdot4871} \times 10^{0\cdot3324}$
$= 10^{0\cdot4871\ +\ \cdot3324}$
$= 10^{0\cdot8195}$
$= \underline{6{\cdot}6}$. Ans.

Example 3.

Divide 7·96 by 5·21.

$7{\cdot}95 \div 5{\cdot}21 = 10^{0\cdot9009} \div 10^{0\cdot7168}$
$= 10^{0\cdot9009\ -\ 0\cdot7168}$
$= 10^{0\cdot1841}$
$= \underline{1{\cdot}528}$. Ans.

EXERCISES 41C

Find the value of the following correct to 4 figures (using logarithm tables):

1. 2·54 × 3·24.
2. 2·87 × 2·31.
3. 4·51 × 1·92.
4. 3·708 × 2·045.
5. 6·123 × 1·132.
6. 5·329 × 1·139.
7. 2·108 × 3·417.
8. 2·073 × 4·107.
9. 8·67 × 1·001.
10. 2·18 × 3·17.
11. 7·569 ÷ 2·321.
12. 8·916 ÷ 3·114.
13. 6·232 ÷ 4·561.
14. 9·819 ÷ 5·321.
15. 9·876 ÷ 5·432.
16. 7·03 × 6·74 ÷ 8·26.
17. 1·78 × 2·01 × 2·12.
18. 8·32 × 1·001 × 1·010.
19. 1·215 × 3·111 × 1·211.

Numbers greater than 10:

Number	Expressed as power of 10			
6·542	$= 6{\cdot}542\times10^{0}$	$=10^{0\cdot8157}$	$\times10^{0}$	$=10^{0\cdot8157}$.
65·42	$= 6{\cdot}542\times10^{1}$	$=10^{0\cdot8157}$	$\times10^{1}$	$=10^{1\cdot8157}$.
654·2	$= 6{\cdot}542\times10^{2}$	$=10^{0\cdot8157}$	$\times10^{2}$	$=10^{2\cdot8157}$.
6542	$= 6{\cdot}542\times10^{3}$	$=10^{0\cdot8157}$	$\times10^{3}$	$=10^{3\cdot8157}$.
65420	$= 6{\cdot}542\times10^{4}$	$=10^{0\cdot8157}$	$\times10^{4}$	$=10^{4\cdot8157}$.
654200	$= 6{\cdot}542\times10^{5}$	$=10^{0\cdot8157}$	$\times10^{5}$	$=10^{5\cdot8157}$.

From the above examples we note

1. So long as the order of the digits of a number remains the same the decimal portion of the logarithm remains the same.

2. The decimal portion is called the MANTISSA.

3. The integral portion of the logarithm is obtained by counting the number of digits before the decimal point and decreasing it by one. If there are four digits before the decimal point then the integral portion of the logarithm is 3. The integral portion is called the CHARACTERISTIC.

For example, in the logarithm of

6·542, the characteristic is 0.
654·2 the characteristic is 2.
6542 the characteristic is 3.
65420 the characteristic is 4.

(One less than the number of digits before the decimal point.)

Exercises (Oral) 41D

1. Write down the characteristics of the logarithms of the following numbers

21; 76; 90; 901; 725; 8·91;
6·23; 2176; 76901; 89543·2; 7200000.

2. Write down the logarithms of the following:

68; 79; 3·156; 4·289; 42·89;
1·111; 200·7; 63847; 7198000;
83,010,000; $25{\cdot}1 \times 10^{2}$; $2{\cdot}561 \times 10^{8}$;
$3{\cdot}196 \times 10^{7}$; $8{\cdot}4 \times 10^{5}$.

Example 4.

Find the value of
729·3 × 8·456.

The Product is

No.	log.
729·3 8·456	2·8629 0·9272
6167	3·7901 (Add)

6167 Ans.

Example 5. Simplify
729·3 ÷ 8·456.

The Quotient is

No.	log.
729·3	2·8629
8·456	0·9272
86·23	1·9357 (Subtract)

86·23 Ans.

Example 6. Evaluate $(27{\cdot}15)^4$

Note the logarithm of 27·15 is multiplied by 4.

$$
\begin{array}{llll}
27{\cdot}15 & = 10^{1\cdot4338} & & \\
\therefore (27{\cdot}15)^4 & = (10^{1\cdot4338})^4 & = 10^{1\cdot4338\times4} & \begin{array}{r} 1{\cdot}4338 \\ 4 \\ \hline \end{array} \\
 & & = 10^{5\cdot7352} & 5{\cdot}7352 \\
 & & = \underline{543500.}\ \text{Ans.} &
\end{array}
$$

Example 7.
Find the value of
$(32{\cdot}45)^3$

By long multiplication, we find the answer to be 34169·931125.

No.	log.
32·45 $(32{\cdot}45)^3$	1·5112 × 3
34160	4·5336

34160 Ans.

Example 8.

Find the value of $\sqrt[3]{528\cdot4}$.

$$\sqrt[3]{528\cdot4}$$
$$= (528\cdot4)^{\frac{1}{3}}.$$

No.	log.
528·4 $(528\cdot4)^{\frac{1}{3}}$	2·7229 2·7229 ÷ 3
8·083	0·9076

8·083 Ans.

EXERCISES 42

Use tables to evaluate the following:

1. 72·9 × 84·56.
2. 365·5 × 2·187.
3. 2·103 × 151·4.
4. 26·65 × 10·11.
5. 793·1 × 3·110.
6. 205·5 × 3·026.
7. 817·5 × 21·76.
8. 425·8 ÷ 3·719.
9. 469·5 ÷ 71·45.
10. 38·19 ÷ 6·001.
11. 89·75 ÷ 11·19.
12. 297·1 ÷ 59·99.
13. $(23\cdot24)^2$.
14. $(10\cdot01)^2$.
15. $(71\cdot56)^3$.
16. $(8005)^3$.
17. $(7\cdot158)^4$.
18. $\sqrt{24\cdot56}$.
19. $\sqrt{189\cdot9}$.
20. $\sqrt{42\cdot56}$.
21. $\sqrt[3]{35\cdot19}$.
22. $\sqrt[3]{78116}$.

23. $\sqrt[3]{878\cdot4}$.

24. $56\cdot19\times48\cdot56\div82\cdot19$.

25. $\sqrt[3]{889\cdot1}\times\sqrt{72\cdot86}\div3\cdot298$.

Positive numbers less than 1.

Example 9.

Find the logarithms of 0·0749, 0·749 and 0·000749.

$$\begin{aligned}0\cdot0749 &= 7\cdot49\div10^{2}\\ &= 10^{0\cdot8745}\div10^{2}\\ &= 10^{0\cdot8745-2}\\ &= 10^{-2+0\cdot8745}=10^{\bar{2}\cdot8745}.\end{aligned}$$

$$\begin{aligned}\text{Similarly } 0\cdot749 &= 7\cdot49\div10\\ &= 10^{0\cdot8745}\div10^{1}\\ &= 10^{-1+\cdot8745}=10^{\bar{1}\cdot8745}.\\ \text{and } 0\cdot000749 &= 7\cdot49\div10^{4}\\ &= 10^{\cdot8745-4}\\ &= 10^{-4+\cdot8745}=10^{\bar{4}\cdot8745}.\end{aligned}$$

So the logarithms are $\bar{2}\cdot8745$, $\bar{1}\cdot8745$ and $\bar{4}\cdot8745$ respectively.

Hence logarithms of numbers between 0 and 1 are negative but are always written so that the DECIMAL portion is POSITIVE. Example: the logarithm of 0·0749 is written as $-2+\cdot8745$ not as $-1\cdot1255$. In practice we write log. 0·0749 as $\bar{2}\cdot8745$.

This is to show that the *characteristic* may be positive or *negative* but the *mantissa* is always *positive*.

The characteristic of the logarithm of numbers between 0 and 1 is obtained by counting the number of digits between the decimal point and where it would be when brought to the standard form of having one digit before the decimal point.

Example 10.

Number	Standard form	Characteristic
0·0749	7·49	−2
0·749	7·49	−1
0·00749	7·49	−3
0·0000749	7·49	−5
0·00000749	7·49	−6

N.B. There is no logarithm of a negative number.

EXERCISES (ORAL) 42A

A. Find the logarithms of the following:

of 729.
of 72·9.
of 7·29.
of 0·729.
of 0·0729.
of 0·00729.

of 0·0001014.
of 0·1213.
of 0·01213.
of 0·02131.
of 0·00001101.
of 0·01201.

B. Find the number whose logarithm is

1. 0·9886.
2. 0·9571.
3. 0·8896.
4. $\bar{1}\cdot7796$.
5. 1·8751.
6. 1·8808.
7. 1·9434.
8. 2·2405.
9. 2·3267.
10. 2·4339.
11. 2·5712.
12. 3·7109.
13. 3·2183.
14. 4·7411.
15. 5·7619.

Example 11.

Evaluate $\sqrt{0\cdot5611}$.

Now, log $\sqrt{0\cdot5611}$ = log $0\cdot5611^{\frac{1}{2}} = \frac{1}{2}$ log 0·5611 $= \frac{1}{2} \times$ log 0·5611·

Number	Log
$\sqrt{0\cdot5611}$	$\bar{1}\cdot7492 \div 2$
	$= \bar{1}\cdot8746$.

$\therefore\ \sqrt{0\cdot5611} = \cdot7492$ (tables).

$= (\bar{1} + \cdot7492) \div 2$
$= (\bar{2} + 1\cdot7492) \div 2$
$= \bar{1} + 0\cdot8746$.

Example 12.

$0\cdot8123 \times 0\cdot02835$

	No.	log.	
	0·8123	$\bar{1}\cdot9098$	Add
	0·02835	$\bar{2}\cdot4526$	
The Product is	·02303	$\bar{2}\cdot3624$	

0·02303 Ans.

Example 13.

$0\cdot8123 \div 0\cdot02835$

	No.	log.	
	0·8123	$\bar{1}\cdot9098$	Subtract
	0·02835	$\bar{2}\cdot4526$	
The Quotient is	28·64	1·4572	

28·64 Ans.

Example 14.

Evaluate (*a*) $(0\cdot8215)^3$.
(*b*) $\sqrt[3]{0\cdot08215}$.

(*a*) No.	logarithm
$(0\cdot8215)^3$	$\bar{1}\cdot9146 \times 3 = \bar{1}\cdot7438$.

$\therefore (0\cdot8215)^3 = 0\cdot5544$.

(*b*) No.	logarithm
$\sqrt[3]{0\cdot08215}$	$\bar{2}\cdot9145 \div 3 = 1\cdot6382$.

$\therefore \sqrt[3]{0\cdot08215} = 0\cdot4347$.

$\bar{2}+0\cdot9145$
$3) = \bar{3}+1\cdot9145$
$-1+0\cdot6382$
$= \bar{1}\cdot6382$.

(*a*) 0·5544
(*b*) 0·4347 } Ans.

EXERCISES 43

Find by logarithm the value of

1. $42{\cdot}5 \times 8{\cdot}127$.
2. $\cdot425 \times 0{\cdot}8127$.
3. $614{\cdot}3 \times 0{\cdot}03617$.
4. $0{\cdot}0003125 \times 44{\cdot}7$.
5. $0{\cdot}01856 \times 0{\cdot}01256$.
6. $0{\cdot}0072 \times 0{\cdot}0089$.
7. $6{\cdot}71 \div 0{\cdot}426$.
8. $3{\cdot}67 \div 429$.
9. $0{\cdot}847 \div 0{\cdot}623$.
10. $1000 \div 0{\cdot}6061$.
11. $1 \div 0{\cdot}7077$.
12. $(0{\cdot}275)^2$.
13. $(0{\cdot}117)^2$.
14. $(0{\cdot}0875)^2$.
15. $(0{\cdot}1234)^3$.
16. $(0{\cdot}3721)^4$.
17. $(0{\cdot}5615)^5$.
18. $\sqrt{0{\cdot}0104}$.
19. $\sqrt{0{\cdot}008}$.
20. $\sqrt{0{\cdot}00072}$.
21. $\sqrt{0{\cdot}001156}$.
22. $\sqrt[3]{0{\cdot}0027}$.
23. $\sqrt[3]{0{\cdot}0072}$.
24. $\sqrt[3]{0{\cdot}01854}$.
25. $\sqrt[4]{0{\cdot}0001267}$.
26. $\sqrt[4]{0{\cdot}01456}$.
27. $\sqrt[4]{0{\cdot}726}$.
28. $(10{\cdot}1)^{\frac{2}{3}}$.
29. $(11{\cdot}18)^{\frac{3}{2}}$.
30. $(12{\cdot}18)^{\frac{3}{5}}$.
31. $\dfrac{42{\cdot}8 \times 5{\cdot}96}{8{\cdot}43}$.

32. $\dfrac{71 \times 82{\cdot}1}{{\cdot}0056}$.

33. $(81{\cdot}96 \div 45{\cdot}62) + 4{\cdot}187$.

34. $\dfrac{\sqrt{71{\cdot}06} \times 182{\cdot}7}{99{\cdot}99}$.

35. $\dfrac{\sqrt{186{\cdot}5} \times 62{\cdot}7}{74{\cdot}95 \times 187{\cdot}6}$.

36. $\dfrac{89{\cdot}4 \times 0{\cdot}0321}{887{\cdot}6 \times 16{\cdot}58}$.

37. $\dfrac{1{\cdot}57 \times 23{\cdot}6 \times 104}{16{\cdot}82 \times 0{\cdot}827 \times 0{\cdot}33}$.

38. $\dfrac{16{\cdot}39 \times \sqrt{0{\cdot}8695}}{(2{\cdot}185)^2}$.

39. $\dfrac{496 \times \sqrt{0{\cdot}7065}}{(11{\cdot}01)^2 \times 0{\cdot}301}$.

EXERCISES 44

Give your answers correct to 3 sig. figures:

1. If $T = 2\pi\sqrt{\dfrac{l}{g}}$ find T when $\pi = 3{\cdot}142$, $l = 25{\cdot}1$ cm and $g = 981$.

2. If $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$, find f when $u = 11{\cdot}21$ cm and $v = 23{\cdot}24$ cm.

3. If $V = \frac{4}{3}\pi r^3$, find V when $r = 3{\cdot}123$ and $\pi = 3{\cdot}142$.

4. If $H = \dfrac{\pi\, dn\, \mathrm{T}}{33000}$, find T when $d = 3{\cdot}7$, $n = 148$ and $H = 16{\cdot}2$.

5. If 1 metre = 39·37 inches, find the height in metres of a mountain 23815 feet high.

6. If $f = \dfrac{uv}{u + v}$, find u when $f = 0{\cdot}69$ and $v = 0{\cdot}89$.

7. If $V = \pi h\,(\mathrm{R} + r)\,(\mathrm{R} - r)$, find its value, given that $\mathrm{R} = 8{\cdot}73$, $r = 7{\cdot}9$ and $h = 2{\cdot}07$.

8. Find the volume of a sphere of radius 6·78 cm from the formula $V = \frac{4}{3}\pi r^3$ where $\pi = 3{\cdot}142$.

9. Simplify (without using tables):

(a) $\log 9 \div \log 3$; (b) $\log 256 \div \log 4$;

(c) $\dfrac{\log\sqrt{12}}{\log 12}$; (d) $\log 5 + \log 20$;

(e) $\log \frac{25}{4} + \log \frac{8}{5}$; (f) $\dfrac{\log 125}{\log 5}$.

10. If $A = P\left(1 + \dfrac{r}{100}\right)^t$, find A,

when $P = £500$, $r = 3\frac{1}{2}$, and $t = 4$.

CHAPTER XVIII

SIMPLE INTEREST—(*Continued from p. 111*)

DISCOUNT

In calculating the interest for a part of a year, say from April 6th to June 4th, we include only one of the dates mentioned. Thus, the number of days from April 6th to June 4th is 59. (See Note on p. 108.)

Example 1.

Rs 7000 is borrowed on April 1st. The rate of interest payable is at first 5%. From the 7th July the rate of interest is changed to 3%, and it remains at that till the principal is repaid on the 10th November. What is the total interest due?

Interest up to 6th July (96 days) = Rs $70 \times 5 \times \frac{96}{365}$.

Interest from 7th July to 10th November (including both days = 127 days) = Rs $70 \times 3 \times \frac{127}{365}$.

$$\text{Total Interest} = \text{Rs } \frac{70}{365}(480 + 381)$$

$$= \text{Rs } \frac{70}{365} \times 861$$

$$= \underline{\underline{\text{Rs } 165{\cdot}12}}. \text{ Ans.}$$

Example 2.

A man deposits Rs 8,000 in a Bank at the beginning of a year. He arranges to withdraw Rs 500 at the end of each month. If the rate of interest is 4%, what interest is due him at the end of the year?

First find the interest due on the full principal Rs 8,000.

Interest = Rs $\frac{8000}{100}\times 4$ = Rs 320. . . . . . (*i*).

Now subtract the interest on:

Rs 500 for 11 months, which is $\frac{500}{100}\times\frac{11}{12}\times 4$
+Rs 500 for 10 months, which is $\frac{500}{100}\times\frac{10}{12}\times 4$
+Rs 500 for 9 months, which is $\frac{500}{100}\times\frac{9}{12}\times 4$

……………………………………………

+Rs 500 for 1 month, which is $\frac{500}{100}\times\frac{1}{12}\times 4$

We have to subtract altogether

Rs $\frac{500}{100}\times\frac{4}{12}(11+10+9+8\ldots+1)$
= Rs $5\times\frac{1}{3}\times 66$
= Rs 110. . . . (*ii*).

The interest due him after a year is

(*i*)−(*ii*) = Rs 320 − Rs 110
= $\underline{\underline{\text{Rs } 210.}}$ Ans.

EXERCISES 45

Find the Simple Interest on:

1. (*a*) Rs 8,000 for the period 5th March to 5th November at 3% per annum.
   (*b*) Rs 75,000 for the period 4th April to 11th October at $4\frac{1}{2}$% per annum.
   (*c*) Rs 9,000 for the period 7th May to 15th December at $3\frac{1}{4}$% per annum.
   (*d*) \$13,000 for the period 23rd April to 16th October at $2\frac{3}{4}$% per annum (nearest cent).
   (*e*) 270,000 fr. for the period 19th May to 1st December at $5\frac{1}{4}$% per annum (nearest cent).

2. Rs 15,000 is borrowed on April 16th. The rate of interest payable is 4% till the 1st June after which it is increased by 1% until the principal is repaid on November 24th. What is the total interest due?

3. £9,500 was borrowed on May 1st. The rate of interest payable was at first fixed at 3%. On July 1st it was lowered to 2% and remained at that till the 17th December when the loan was repaid in full. What interest was due?

4. $7,600 was borrowed on April 1st at 4% interest. On June 6th the rate of interest was enhanced to 5% and on October 1st it was reduced to 3%. What was the interest due by December 1st when the loan was repaid?

5. A man deposits Rs 15,000 in a Bank at the beginning of a year at 3% S.I. He arranges to withdraw Rs 1,000 at the end of each month. What interest is due him at the end of the year?

6. A man's deposit account at the beginning of a year stood at £6,000. He withdrew £200 at the end of each month. If Bank rate of interest was 2½%, what interest was due him after 6 months?

7. A man's deposit account at the beginning of a year stands at 170,000 fr. If he adds 50,000 fr. to the Principal at the end of each month, what interest, the rate being 4½%, will be due him at the end of the year?

8. If Rs 2,000 amounts to Rs 2,422·50 in 6½ years, what will be the interest on Rs 3,500 for 5 years at the same rate?

9. A man divided his Principal of Rs 10,000 into two parts. One part he deposited at 3½% interest, the other part he deposited at 4⅓% interest. At the end of the year he received an interest of Rs 426 on the combined investments. How did he divide his Principal? (Use algebraic symbols.)

BANK DISCOUNT

Suppose Robinson and Co., merchants, placed an order for goods with another merchant, G. Smith. One way in which they could arrange for payment is by a Bill of Exchange. If the goods cost Rs 750, Smith will draw up a Bill of Exchange in some such form as this:

Bill of Exchange

---

54 Merton Avenue,
Bombay, June 4th, 1963.

Rs 750.

Three months after date pay to me or to my order the sum of Rupees Seven hundred and fifty for value received.

G. Smith

To

Messrs Robinson and Co.,
105 Heston Avenue,
Calcutta.

Here Smith is the **drawer** of the bill. The bill has been drawn on Robinson and Co. They are the **drawees.** If the drawees find the bill in order, they accept it. **Acceptance** is made by the drawees writing across the face of the bill:

Accepted,
Robinson and Co.,
Calcutta, June 11th, 1963.

The Bill is properly stamped according to the sum being negotiated, and returned to the drawer.

Since G. Smith has made the bill payable to himself, he is the **payee.** He could, however, have drafted the bill so as to make it payable to (say) Jones and Co., in which case Jones and Co., would be the **payees.**

The Acceptors are then legally responsible for payment when the bill *matures*, that is on September 4th, that is, 3 months after June 4th.

The 'three months' mentioned in the bill is its term or currency. A bill may also be drafted for any other term or currency.

The term or currency gives Robinson and Co. time to find the money for payment from the sale of some or all of the goods.

Bills of Exchange are thus instruments for facilitating and promoting business. If Robinson and Co. were not allowed time for payment of the goods, they would probably not be able to buy and G. Smith would lose a customer.

G. Smith himself need not wait the three months for his money. He may endorse the bill to his bank, who will take over charge of it and pay him the **present value** of the bill.

Suppose G. Smith takes the bill to his bank on July 4th. The bill is not due to mature till the 4th September, which is two months later. The bank will subtract two months' interest from the value of the bill. Thus, if the rate of interest is 5 per cent. they will subtract

$\frac{2}{12}$ of $\frac{5}{100}$ of Rs 750, that is Rs 6·25.

Rs 6·25 is called the **discount.**

The Bank will pay G. Smith Rs 750 – Rs 6·25 or Rs 743·75. Rs. 743·75 is the **present** or **discounted value** of the bill.

The bill now belongs to the bank. It will recover the full value of Rs 750 from Robinson and Co.'s bank when the bill matures on September 4th.

There is an important point of which we must take note in the above calculation. The bank has treated the Rs 750 as the principal and calculated the interest on it. Strictly speaking Rs 750 is the amount *after* 2 months and they should pay G. Smith that principal which would amount to Rs 750 in 2 months at 5 per cent. If they were to do this, the calculation would be as follows:

Int. on Rs 100 for 2 months at 5 per cent $= \frac{2}{12}$ of Rs 5 $=$ Rs $\frac{5}{6}$.

So, Rs 100 amounts to Rs $100\frac{5}{6}$;

$\therefore$ the sum that amounts to Rs 750

$$\text{is Rs } \frac{750}{100\frac{5}{6}} \times 100, \text{ that is Rs } \frac{750 \times 6}{605} \times 100$$

$$= \text{Rs } 743{\cdot}80.$$

Rs 743·80 is the true present or discounted value of Rs 750.

The **true** or **arithmetical** discount

$= $ Rs 750 – Rs 743·80 $=$ Rs 6·20.

Banks, however, do not calculate the **true discount.** They calculate in the first way shown above. The discount thus found is called banker's discount.

The discount is really what the bank takes for its trouble, and the discounting of bills is one of the important functions of a bank.

Which is greater, true discount or banker's discount?

A bill does not mature legally till 3 days after the stated date. These days are spoken of as *days of grace*. In actual practice, therefore, the bank adds on 3 days in reckoning the period left for maturation.

The example below illustrates this.

Example 3.

Find correct to the nearest nP. the discounted value of a bill for Rs 600 drawn on May 4th for 3 months and discounted on May 13th at 6 per cent.

The bill is due to mature on August 4th. The bank reckons the period left for maturation as from May 13th to August 7th, that is, 86 days.

The interest on Rs 600 for 86 days at 6 per cent is Rs $600 \times \frac{86}{365} \times \frac{6}{100}$, that is, Rs 8·482.

This is the discount to be deducted from Rs 600.

The discounted value of Rs 600 is Rs 591·518.
that is, Rs 591·52 to the nearest nP. Ans.

Example 4.

A bill for Rs 500, drawn on April 20th for 3 months was discounted on April 27th, for Rs 490·47. What was the rate of discount?

The bill was due to mature on July 20.

On April 27 it had 84 days to mature.

With the 3 days of grace added, the time is 87 days.

The question now is: At what rate will Rs 500 yield an interest of Rs 9·53 in 87 days? We know $I = \frac{PRY}{100}$ or $R = I \times \frac{100}{PY}$.

∴ The rate

$$= \frac{\text{Rs } 9{\cdot}53}{\text{Rs } 500} \times \frac{365}{87} \times 100 \text{ per cent}$$

$$= \frac{9{\cdot}53}{5} \times \frac{365}{87} \text{ per cent} = \frac{9{\cdot}53 \times 73}{87} \text{ per cent} = 8 \text{ per cent.}$$

(The simplification must not be expected to work out to *exactly* 8 per cent, as the discounted value is not exact, but calculated to the nearest nP.)

EXERCISES 46

1. Find to the nearest nP. the discounted value of these bills. Three days' grace must be allowed in each case.

(*a*) Rs 750 drawn on August 1 for 3 months and discounted on August 20, at 4 per cent.

(*b*) Rs 1,000 drawn on April 6 for 6 months and discounted on May 1 at 5 per cent.

(*c*) Rs 940 drawn on May 15 for 1 month and discounted on May 20, at 4½ per cent.

(*d*) Rs 1,100 drawn on May 15 for 3 months and discounted on May 20, at 4½ per cent.

(*e*) Rs 2,050 drawn on April 10 for 60 days and discounted on April 20, at 6 per cent.

(*f*) Rs 1,750 drawn on May 7 for 90 days and discounted on May 21, at 5 per cent.

2. Find to the nearest penny the discounted values of these bills. Three days' grace must be allowed in each case.

(*a*) £1,000 drawn on April 10 for 3 months and discounted 10 days later at 5 per cent.

(*b*) £1,050 drawn on April 10 for 60 days and discounted on April 20 at 6 per cent.

(*c*) £9,800 drawn on May 2 for 3 months and discounted on May 12 at 6 per cent.

(*d*) £8,900 drawn on May 2 for 90 days and discounted on May 12 at 5 per cent.

3. A bill for Rs 700 drawn on March 22 for 3 months was discounted on April 13 for Rs 693. What was the rate of discount? (Allow 3 days' grace.)

4. A bill for Rs 1000 drawn on May 3 for 60 days was discounted on May 12 for Rs 991·75. What was the rate of discount? (Allow 3 days' grace.)

5. A bill for £1,200 drawn on June 4 for 90 days was discounted on June 11 for £1187 8*s*. 5*d*. What was the rate of discount? (Allow 3 days' grace.)

6. A bill for £800 drawn on June 4th for 6 months was discounted on June 20 for £773 18*s*. 4*d*. What was the rate of discount?

CHAPTER XIX

COMPOUND INTEREST

When money is deposited at Simple Interest the interest is drawn by the depositor as it becomes due. If, however, the interest is not so drawn but left in the bank, it may be added to the principal at the end of the period stipulated and the combined amount then earns further interest. Such interest is called Compound Interest.

Suppose, for example, Rs. 700 is deposited at Simple Interest for 2 years at 4%. At the end of the first year the interest due, viz., Rs $\frac{700 \times 4}{100}$ = Rs 28, may be drawn by the depositor. The principal remains Rs 700, and at the end of the second year the depositor will earn another Rs 28. In the two years he will earn altogether Rs 56.

If, however, the Rs 700 is deposited at Compound Interest, the interest of Rs. 28 will not be paid to the depositor at the end of the year, but it will be added to the principal normally at the end of the year. The principal for the second year will be Rs 728 and it is on this sum that the interest for the second year must be calculated.

The interest for the second year will be $\frac{4}{100}$ of Rs 728 i.e. Rs 29·12 and not Rs 28 as was the case with Simple Interest.

We see that at Compound Interest the interest earned over two years will be Rs 28+Rs 29·12=Rs 57·12 as compared with Rs 56 in the case of Simple Interest.

In calculating Compound Interest we may set out our work systematically as shown in the Example below.

Example 1.

Find the Compound Interest on Rs 7550 for 3 years at 4%.

	Rs		
	7550	Principal (P)	for 1st year
Stage 1	(75·50	1% of P)	
	302·00	4% of P=interest for 1st year	
	7852·00	Principal (new P) for 2nd year	

B/F	7852·00	Principal (P) for 2nd year
Stage 2	(78·52	1% of new P)
	314·08	4% of new P=interest for 2nd year
	8166·08	Principal (newer P) for 3rd year
Stage 3	(81·6608	1% of newer P)
	326·6432	4% of ,, P=interest for 3rd year
	8492·7232	Amount at the end of 3rd year
Subtract	7550	Original Principal
Rs	942·7232	Compound Interest for the 3 years

The C. interest is Rs 942·72 to the nearest nP. *Ans.*

NOTE. At each stage we add the interest to the principal to find the amount which is then taken as the principal for the following stage. At the end of the 3rd stage we subtract the original principal from the amount. The difference is the interest payable. At a later stage, we may do the 1% line mentally and multiply by the rate direct. In this case multiply by 4 and write the figures two places to the right.

When we are dealing in a non-decimal money system (such as £ *s d*) it is usually better to first decimalize the principal. The interest is then calculated as shown below and the result re-converted.

Example 2.

Find the C.I. on £ 7,165 10*s*. 6*d*. for 2 years at 6%.

	£	*s.*	*d.*
	7165	10	6
=	7165	10·5	

= 7165·525	principal for 1st year
(71·65525	1% of P)
429·93150	6% of P=int. for 1st year
7595·4565	principal for 2nd year
(75·954565	1% of new P)
455·727390	6% of new P=int. for 2nd year
£ 8051·183890	amount at the end of 2nd year

£ 8051·183890
s. 3·6778
d. 8·1336 ∴ Amount = £8051 3*s.* 8*d.*
Original Principal = £7165 10*s.* 6*d.*
C.I. = £ 885 13*s.* 2*d.* Ans.

A somewhat different procedure from that shown in the previous examples can be adopted for the calculation of Compound Interest. This procedure is to calculate the sum to which Re 1 will amount at the given rate of interest for the given period of time and then to multiply the result by the Principal.

Thus to calculate the Compound Interest at 2% for 3 years.

At 2% the interest on Re 1 for 1 year = Re 0·02.

Re 1 will therefore amount to Rs 1·02 at the end of the first year.

Now we see that to obtain the amount at the end of the first year, we multiply the principal at the beginning of the year by the factor 1·02.

Since 1·02 is the principal for the second year,
$1{\cdot}02\times1{\cdot}02$ or $(1{\cdot}02)^2$ will be the amount at the end of 2nd year.
Similarly $(1.02)^3$ will be the amount at the end of 3rd year, and so on.

We now argue:

Since Re 1 will amount to Rs $(1{\cdot}02)^3$ in 3 years.
the principal Rs P will amount to Rs $P\times(1{\cdot}02)^3$.

1·02 in this case is called the growth factor. If the rate of interest is 5%, then 1·05 is the growth factor. If the rate is 7% then 1.07 is the growth factor and so on. If r is the rate of interest, the growth factor is $1+\frac{r}{100}$, and we write our formula for the amount (A) as

$$A=\left(1+\frac{r}{100}\right)^n,$$ where n is the number of years.

(See p. 137)

Example 3.

Find the C. I. on Rs 5000 for 2 years at 3%.

$$\begin{aligned}\text{The amount (A)} &= P\times\left(1+\frac{r}{100}\right)^n\\ &= \text{Rs } 5000\times(1{\cdot}03)^2\\ &= \text{Rs } 5000\times1{\cdot}0609\\ &= \text{Rs } 5304{\cdot}50\\ \text{The interest} &= \text{Rs } 5304{\cdot}50-\text{Rs } 5000\\ &= \text{Rs } 304{\cdot}50.\end{aligned}$$

Compound Interest Tables save on the work of calculating the power of the growth factor. Thus if we have to calculate the amount at 2% for 3 years, we can save ourselves the trouble of calculating $(1{\cdot}02)^3$, by looking up Tables such as those given below. Under the column for 2% in the row for 3 years we find the required power of the growth factor to be given as 1·0612080. (This is verified here.) To find the amount we multiply the principal by 1·0612080:

$$\begin{array}{r}
1{\cdot}02 \\
1{\cdot}02 \\ \hline
1{\cdot}02 \\
204 \\ \hline
1{\cdot}0404 \\ \hline
1{\cdot}02 \\
1{\cdot}0404 \\ \hline
1{\cdot}04040 \\
0{\cdot}020808 \\ \hline
1{\cdot}061208
\end{array}$$

Tables of Powers of Growth Factors

n	1%(R=1·01)	2%(R=1·02)	3%(R=1·03)	4%(R=1·04)
1.	1·0100000	1·0200000	1·0300000	1·0400000
2.	1·0201000	1·0404000	1·0609000	1·0811600
3.	1·0303010	1·0612080	1·0927270	1·1248640
4.	1·0406040	1·0824322	1·1255088	1·1698586

(See p. 138.)

EXERCISES 47

Find the C.I. on (using the Method of Example 1):

1. Rs 3000 for 2 years at 3%
2. Rs 5000 for 3 years at 2%
3. Rs 7100 for 3 years at 3%
4. Rs 8500 for 4 years at 2%
5. Rs 6500 for 3 years at 4%
6. Rs 12000 for 3 years at 5%
7. Rs 15000 for 2 years at 10%
8. Rs 20000 for 3 years at 3%

(using the Growth Factor Tables):

9. Rs 5000 for 3 years at 2%
10. Rs 7000 for 3 years at 3%
11. Rs 8000 for 2 years at 3%
12. Rs 6000 for 4 years at 2%
13. Rs 5000 for 3 years at 4%
14. Rs 5000 for 4 years at 3%.

Example 4

Find the C. I. on Rs 7500 for 3 years at $4\frac{5}{8}\%$. What is the difference between the S.I. and the C.I. during the period mentioned?

(*i*) C. I.	Rs		
	7500	Principal for 1st year.	
	(75·00	$\frac{1}{100}$ of P.)	
	300·00	at 4%	Int. for 1st year
	37·50	at $\frac{1}{2}\%$	
	9·375	at $\frac{1}{8}\%$	
	7846·875	Principal for 2nd year	
	(78·46875	$\frac{1}{100}$ of new P.)	
	313·87500	at 4%	Int. for 2nd year
	39·23437	at $\frac{1}{2}\%$	
	9·80859	at $\frac{1}{8}\%$	
	8209·79296	Principal for 3rd year	
	(82·09793	$\frac{1}{100}$ of newer P.)	
	328·39172	at 4%	Int. for 3rd year
	41·04896	at $\frac{1}{2}\%$	
	10·26224	at $\frac{1}{8}\%$	
	Rs 8589·49588	Amount at the end of 3 years	
	7500·00000	Original Principal	
	1089·49588	C.I. for 3 years	
	=Rs 1089·50	to the nearest nP.	

(*ii*) $\text{S.I.} = \text{Rs}\ \frac{7500\times3}{100}\times4\frac{5}{8} = \text{Rs}\ 1040{\cdot}63.$

The difference between C. I. and S. I. is Rs 48·87 Ans.

Example 5.

What is the C. I. on Rs 75650 for 2 years at 2% if the interest is calculated and added half-yearly?

The amount would be the same as if the interest were calculated at 1% for 4 years.

Work the sum out on this assumption and see that you obtain the answer Rs 3071·69.

EXERCISES 48

1. Find the Compound Interest on:

(*a*) Rs 6300 for 3 years at 4% (nearest nP.)
(*b*) Rs 7500 for 3 years at $3\frac{1}{3}\%$ (nearest nP.)
(*c*) Rs 8800 for 2 years at $3\frac{3}{4}\%$ (nearest nP.)
(*d*) Rs 9500 for $2\frac{1}{2}$ years at $3\frac{5}{8}\%$ (nearest nP.)
(*e*) £7568 for 2 years at 4% (nearest penny).
(*f*) £8935 for $3\frac{1}{2}$ years at $4\frac{1}{4}\%$ (nearest penny).
(*g*) 37835 fr. for 2 years at $2\frac{5}{8}\%$ (nearest fr.)
(*h*) 93125 fr. for $2\frac{1}{2}$ years at $3\frac{1}{4}\%$ (nearest cent).
(*i*) $8300 for 3 years at 4% (nearest cent).
(*j*) $9538 for $2\frac{1}{2}$ years at $2\frac{7}{8}\%$ (nearest cent).

2. In each of the following cases find the difference between the C. I. and the S. I. for the period given (answer to nearest nP., penny or cent):

(*a*) £7315 15*s*. for 3 years at $2\frac{5}{8}\%$.
(*b*) Rs 8634.50 for 3 years at $3\frac{1}{4}\%$.
(*c*) Rs 73,565 for 4 years at 2%.
(*d*) Rs 9678.62 for 2 years at $3\frac{3}{4}\%$.
(*e*) 73856 fr 40 cents for $2\frac{1}{2}$ years at 4%.
(*f*) $89370 for $3\frac{1}{2}$ years at $3\frac{5}{8}\%$.

3. Find the C. I. in the following cases, if the interest is added half-yearly. In such case calculate how much additional interest accrues through a half-yearly instead of a yearly addition of interest.

(*a*) 7936 for 2 years at 3% (nearest nP.)
(*b*) Rs 10500 for 2 years at 3% (nearest nP.)
(*c*) £7615 13*s*. 4*d*. for $2\frac{1}{2}$ years at 2% (nearest penny).

The previous procedure for the calculation of compound interest has been set out in a manner that will enable the student to grasp step by step the nature of the increments of interest involved.

That procedure, however, is suitable in practice only when the period of the loan is limited to two or three years and the rate is integral. Otherwise, the method explained below will be found to be more rapid and convenient, since logarithms can be used.

Compound Interest Tables have also been compiled. They are, of course, used in commercial practice; and the student should know how to read them and use them when they will facilitate his work.

He is required, however, to know how to calculate the C.I. without the use of the C.I. Tables and incidentally to understand how the Tables themselves are calculated and compiled.

(See p. 133.)

Suppose a sum of money is placed at C.I. and the rate of interest per annum is 3 per cent.

In the first year the interest on Re 1 will be Re 0·03 so that, at the end of the 1st year, Re 1 will amount to Rs 1·03. From this we infer that the amount at the end of the year can be obtained from the principal at the beginning of the year by multiplying the latter by the factor 1·03. Thus Rs 1·03 is the principal for the second year.

Repeating the argument for the second year, we infer that since the principal for that year is Rs 1.03, the amount at the end of the second year will be Rs $1{\cdot}03 \times 1{\cdot}03$, that is Rs $(1{\cdot}03)^2$.

Carrying on the argument we conclude that Re 1 will amount

at the end of the 1st year to Rs 1·03
at the end of the 2nd year to Rs $(1{\cdot}03)^2$
at the end of the 3rd year to Rs $(1{\cdot}03)^3$
at the end of the 4th year to Rs $(1{\cdot}03)^4$
and so on.

At the end of n years Re 1 will amount to Rs $(1{\cdot}03)^n$. If we denote the rate by r, then we can say that

Re 1 will amount to Rs $(1+\frac{r}{100})^n$ in n years.

∴ the amount A for principal of P will be given by the formula

$$A=P\left(1+\frac{r}{100}\right)^n$$

or $A=PR^n$, where R stands for $(1+\frac{r}{100})$,

R or $(1+\frac{r}{100})$ being called the growth factor.

The Amount at C. I., as has just been seen, is found by multiplying the Principal by the nth power of the growth factor where n stands for the number of years.

An extract from C.I. Tables is given below. Opposite the number of years there and below the figure specifying the rate of C.I. is given the required power of the growth factor. Thus opposite 3 years and below 2% is given 1·0612080. This approximately equals $(1+\frac{r}{100})^3$ and represents the Amount on Re 1 at the end of 3 years at 2% C.I. The amount on a principal of Rs P will be equal to Rs P $(1+\frac{2}{100})^3$ or Rs P×1.0612080.

Table showing amount of Re 1 compound interest.

(See p. 134.)

Years	$1\frac{1}{2}\%$	2%	$2\frac{1}{2}\%$	3%	Years
1	1·0150000	1·0200000	1·0250000	1·0300000	1
2	1·0302250	1·0404000	1·0506250	1·0609000	2
3	1·0456784	1·0612080	1·0768906	1·0927270	3
4	1·0613636	1·0824322	1·1038129	1·1255088	4
5	1·0772840	1·1040808	1·1314082	1·1592741	5
6	1·0934433	1·1261624	1·1596934	1·1940523	6
7	1·1094449	1·1486857	1·1886858	1·2298739	7
8	1·1264926	1·1716594	1·2184029	1·2667701	8
9	1·1433900	1·1950926	1·2488630	1·3047732	9
10	1·1605408	1·2189944	1·2800845	1·3439164	10

(P.T.O.)

(Contd.)

Years	$3\frac{1}{2}\%$	4%	$4\frac{1}{2}\%$	5%	Years
1	1·0350000	1·0400000	1·0450000	1·0500000	1
2	1·0712250	1·0816000	1·0920250	1·1025000	2
3	1·1087179	1·1248640	1·1411661	1·1576250	3
4	1·1475230	1·1698586	1·1925186	1·2155062	4
5	1·1876863	1·2166529	1·2461819	1·2762816	5
6	1·2292553	1·2653190	1·3022601	1·3400956	6
7	1·2722793	1·3159318	1·3608618	1·4071004	7
8	1·3168090	1·3685691	1·4221006	1·4774554	8
9	1·3628974	1·4233118	1·4860951	1·5513282	9
10	1·4105988	1·4802443	1·5529694	1·6288946	10

Example 6.

Find the C.I. on Rs 6300 for 3 years at 4%.

We look up the C.I. tables opposite 3 years and below 4% and obtain 1·1248640.

This number (in Rs) is the amount on a principal of Re 1 at the given rate in the given time.

To obtain the amount on the given principal we multiply the given principal by this number.

The amount = Rs $P\times1{\cdot}1248640$
= Rs $6300\times1{\cdot}1248640$
= Rs 7086·64 to the nearest nP.
The principal is Rs 6300
∴ The C.I. is Rs 786·64 Ans.

Example 7.

Calculate, without the use of C.I. tables, the amount on Rs 7500 for 20 years at 3·5%.

We use the formula $A=PR^n$.

$\therefore\ A=7500\times(1{\cdot}035)^{20}$

(in Rupees)

Number	logarithm
1·035	0·0149403
$(1{\cdot}035)^{20}$	0·2988060
7500	3·8750613
14923	4·1738673

The amount $=$ Rs 14933. Ans.
The principal $=$ Rs 7500.
$\therefore$ The C.I. $=$ Rs 7423.

The student should now work Exercises 48 No. 1 both with and without the use of C.I. tables.

CHAPTER XX

HARDER PERCENTAGES

(*Note carefully how in working out the problems the absolute units are used.*)

Example 1.

The length of a rectangle is increased by 25%, and the width reduced by 12%. How is the area affected?

A (previously) $=l\times b$.

A (now) $=\frac{125}{100}l\times\frac{88}{100}b$

$$=\frac{125\times 88}{100\times 100}\text{ of } l.b.$$

$$=\frac{125\times 88}{100}\text{ \% of } l.b.$$

$$=\frac{125\times 88}{100}\text{ \% of what it was}$$

$$=110\text{ \% of what it was}$$

So the area has been increased by 10%. Ans.

Example 2.

By how much per cent must a motorist increase his speed in order to reduce the time taken to do a certain distance by 12%?

Distance done $=v\times t$ (speed $\times$ time) (*i*)

$=$new speed $\times\frac{88}{100}t$ (*ii*)

If (*i*) is to be the same as (*ii*),

then new speed $=\frac{100}{88}$ of previous speed

$=\frac{100}{88}\times 100\%$ of previous speed

$=113\frac{7}{11}\%$ of previous speed.

$\therefore$ the speed must be increased by $13\frac{7}{11}\%$. Ans.

Example 3.

Two men's wages are in the ratio 4:5. If the first man's wages are increased by 10% and the second's by 12%, in what ratio is the total payment increased?

Let the men receive at first $4x$ and $5x$ respectively. After the rise they receive $\frac{110}{100}$ of $4x$ and $\frac{112}{100}$ of $5x$ respectively.

The previous total was $9x$.

The later total is $\frac{110}{100}$ of $4x+\frac{112}{100}$ of $5x=10x$.

The increase is, therefore, equal to $10x-9x=x$.

Hence the percentage increase is $\frac{x}{9x}\times 100\%$, that is $11\frac{1}{9}\%$.

$11\frac{1}{9}\%$ Ans.

EXERCISES 49

1. A merchant A made a profit of $33\frac{1}{3}\%$ on his outlay when he sold goods to an agent B. B in his turn sold the goods to a customer C. Had A sold the goods direct to C at the price which C paid, he (A) would have made a profit of 50%. What percentage profit did B make?

2. A dealer ordinarily makes a profit of 15%. If his cost goes down 20% and he lowers his price by 10%, what percentage profit will he make?

3. In 1957 a man sold 90 articles at Rs 3·50 each, making thereby a profit of 12% on what he paid for them. The following year when the price of the articles had risen by 25 nP. each, he bought another 90 articles. Fifty of these he sold at the same price as before, but the remainder at 25 nP. each more. What was his percentage profit in 1958?

4. A dealer retails goods at 8*d.* a lb. thereby making a profit of 8% on his outlay. What is his cost price per cent (nearest penny)?

5. What profit does a shopkeeper make if he marks his prices at 12% over cost and then allows a cash discount of 5% off the marked price?

6. A used-car dealer bought two cars at a total cost of Rs 3500. He sold one of them for Rs 2600, making on it a profit of 30%. The other he sold at a profit of 25%. What percentage profit did he make on the whole?

7. Two men's wages are in the ratio 3 : 4. If one of them is given a rise of 20% and the other a rise of 25%, in what ratio will their wages be?

8. The populations of two towns are in the ratio 5 : 7. If the population of the first goes up 26%, while the other falls 25%, in what ratio will the populations then be?

9. After lowering his selling price by $12\frac{1}{2}$%, a dealer finds that his profit on 120 articles is reduced by £30. What was the original selling price of an article?

10. A dealer who used to make a profit of 30% on his cost price, raised his selling price by 12%. This had the effect of decreasing by 8% the number of articles he was able to sell in a year. What percentage profit did he make during the year in question? (Answer to nearest unit.)

11. Two districts arrange for an exchange of inhabitants. The first sends 1250 to the second, and the second sends 2300 to the first. After the exchange it is found that the population of the first district has gone up 3%, while that of the second has fallen 2%. What were the previous populations of the districts?

12. A manufacturer fixes his price at 33% over the cost of production. If the cost of production goes up 12% and the manufacturer raises his price by 10%, what will his percentage profit be?

13. An orange dealer fixes his price at 30% above cost. If 75 oranges out of 1000 go rotten and are unsaleable, by how much will his percentage profit fall?

14. A cycle dealer fixes his selling price so as to make a profit of 15% on his outlay. What percentage profit will he make if he reduces his selling price by 5%?

15. A manufacturer's total costs of manufacture work out at £383 6*s*. 8*d*. per thousand articles. The cost of advertisement, which he reckons separately, works out at 8*d*. per article. If he fixes his selling price at 25% above the actual cost of manufacture, what is his actual percentage profit?

16. A sweepstake committee fixes the first prize at 40% of what remains of the takings from the sale of tickets after 15%

has been deducted for expenses. How many tickets, at Re 1 each, must be sold for the first prize to be Rs 10200 ?

17. A coal merchant buys a wagon-load of coal for Rs 337. After paying freight at Rs 3 per ton he sells out the whole amount at Rs 21 a ton and makes thereby a profit of $10\frac{1}{4}$% on his outlay. How many tons were there to the wagon-load?

18. A railway company lowers the cost of the ticket between two places by 8%. If the number of passengers who book between these places increases by 12%, what will be the percentage change in the railway income for this particular journey?

19. The length of the side of a square is increased 8%. By how much per cent is its area increased?

20. The radius of a circle is increased 14%. By how much per cent is its area increased?

21. The length of a rectangle is increased 5% and its breadth 4%. What is the percentage increase in the area?

22. The length of a rectangle is increased 5% while its breadth is decreased 5%. What is the percentage change in the area?

23. The side of a cube is increased 8%. What is the percentage change in its volume? What is the percentage change in its surface?

24. The sides of a rectangular solid are increased 7%, 8%, 10% respectively. What is the percentage increase in the volume?

25. The length of a rectangular solid is increased by 12% the width is increased by 10%, and the depth is decreased by 20%. What is the percentage change in the volume?

26. What is the percentage change in the volume of a spherical bubble when its diameter expands by 15%?

27. By how much per cent must a motorist increase his speed in order to reduce by 20% the time taken to do a certain distance?

28. By how much per cent must a motorist reduce his speed to increase by 25% the time taken to do a certain distance?

29. If a motorist increases his speed 20%, what percentage change will there be in the time taken to do a certain distance?

30. If a motorist reduces his speed 11%, what percentage change will there be in the time taken to do a certain distance?

31. The length of a rectangular solid is to be increased 21% and its breadth is to be increased 11%, while its volume is to remain constant. By how much per cent must its depth be decreased?

32. A rectangular piece of metal sheeting costs Rs 32. The cost of metal sheeting of a better quality is 15% more than this. What is the cost of a piece of the latter sheeting which is 10% longer and 25% wider than the first piece?

CHAPTER XXI

MIXTURES INVERSE PROBLEMS

ADDING IN ADULTERANT WHICH COSTS NOTHING

Example 1.

If milk which costs 75 nP. litre is mixed with water costing nothing in the ratio of 1 of water to 5 of milk, then every 6 litres of the mixture will cost 5×75 nP., *not* 6×75 nP.

So, we see, the cost price of the mixture is $\frac{5}{8}$ of the cost price of 6 litres of pure milk.

Example 2.

Suppose spirits are mixed with water in the ratio of 1 of water to 7 of spirits. The cost price of 8 gallons of the mixture will be the same as the cost piece of 7 gallons of spirits. That is, the cost price of the mixture is $\frac{7}{8}$ of the cost price of 8 gallons of spirits.

Conversely: If the cost price of a mixture of a pure product and an adulterant which costs nothing is $\frac{7}{8}$ of the cost price of 8 gallons of the pure product, we can conclude that 7 parts of the pure product have been mixed with one part of the adulterant.

Also: if the cost price of the mixture is $\frac{4}{9}$ of the cost price of 9 parts of the pure product, we can conclude that 4 parts of the pure product have been mixed with 5 parts of the adulterant.

Example 3.

In what ratio must spirit costing Rs 9·50 a litre be diluted with water, so as to bring the cost price down to Rs 8·75 a litre?

The cost price of the mixture is Rs $8\frac{3}{4}$.
The cost price of the pure spirit is Rs $9\frac{1}{2}$.

The ratio of the price is $\frac{8\frac{3}{4}}{9\frac{1}{2}}$ or $\frac{35}{38}$.

So the ratio is 3 units of water to 35 units of spirit.

Algebraic Solution

Let x litre of spirit be mixed with 1 litre of water.

x litre of spirit costs Rs $\frac{19}{2}x$.

1 litre of water costs nothing.

So (adding), $(x+1)$ litres of the mixture cost Rs $\frac{19}{2}x$.

But $(x+1)$ litres cost Rs $8\frac{3}{4}(x+1)$.

$$\therefore \frac{35}{4}(x+1) = \frac{19}{2}x$$

$$\text{or } 35(x+1) = 38x$$

$$\text{or} \qquad x = \frac{35}{3}.$$

$\frac{35}{3}$ litres of spirit are mixed with 1 litre of water, or 35 litres of spirit with 3 litres of water.

MIXING AND BLENDING

Example 4.

In what ratio must coffee costing Rs 3·60 a lb. be mixed with chicory costing Rs 1·40 a lb. in order that the mixture will cost Rs 3 a lb.?

Mixture
Rs 3 a kg

Coffee	*Chicory*
Rs 3·60 a kg	Rs 1·40 a kg
'loss' of Rs 0·60 a kg	'gain' of Rs 1·60 a kg

In order that the 'gain' shall balance the 'loss', 160 parts of coffee must be mixed with 60 parts of chicory.

The ratio is, therefore, 160 : 60,
that is 8 : 3.

Algebraic Solution

Let the ratio be x of coffee to 1 of chicory.

$\therefore$ x kg of coffee costs Rs $3{\cdot}60x$

1 kg of chicory costs Rs $1{\cdot}40$

$(x+1)$ kg of the mixture costs Rs $(3{\cdot}60x+1{\cdot}40)$

but $(x+1)$ kg of the mixture should cost Rs $3(x+1)$;

$$\therefore\ 3{\cdot}60x+1{\cdot}40=3x+3$$

$$\therefore\ 0{\cdot}60x=1{\cdot}60$$

$$\therefore\ x=\frac{1{\cdot}60}{0{\cdot}60}=\frac{8}{3}$$

The ratio is, therefore, 8 : 3.

INTRODUCING CONSIDERATIONS OF PERCENTAGE PROFIT AND LOSS

Example 5.

In what ratio must teas costing 2*s*. 6*d*. a kg and 1*s*. 10*d*. a kg be mixed, in order that by selling the blend at 2*s*. 4*d*. a kg a profit of 12% will be made?

We must first arrive at the cost price of the mixture.

The cost price $\frac{100}{112}$ of 2*s*. 4*d*. $=\frac{100}{112}\times 28d.=25d.$

Now continue as in Example 4.

The required ratio is 3 : 5.

Example 6.

Work Example 3 by the method of Example 4.

Pure Spirit
Rs $9\frac{1}{2}$ a l

Water
Re 0

Mixed Product
Rs $8\frac{3}{4}$.

The loss in one case is Rs $(9\frac{1}{2}-8\frac{3}{4})=$ Rs $\frac{3}{4}$ a l.

The gain in the other case is Rs $(8\frac{3}{4}-0)=$ Rs $8\frac{3}{4}$ a l.

To counterbalance gain and loss we shall have to mix pure spirit and water in the ratio $8\frac{3}{4}:\frac{3}{4}$ that is, 35 : 3.

Example 7.

In what ratio must three liquids, containing 24%, 32%

and 18% alcohol respectively, be mixed for the mixture to contain 22% alcohol?

$$\left.\begin{array}{lll}\text{The 1st liquid contains} & 2\% \textit{ more} & \text{alcohol}\\ \text{The 2nd} \quad ,, \quad ,, & 10\% \textit{ more} & ,,\\ \text{The 3rd} \quad ,, \quad ,, & 4\% \textit{ less} & ,,\end{array}\right\}\text{than the mixture.}$$

So that, taking x parts of the 1st liquid, y parts of the 2nd, and z parts of the 3rd,

$2x+10y=4z$ if the 'gains' are to counterbalance the 'losses'.

An infinite number of sets of values will satisfy this equation, giving an infinite number of answers to our problem.

Thus if $z=5$ y can be $=1$ and $x=5$;
or if $z=4$ y can be $=1$ and $x=3$;
or if $z=7$ y can be $=2$ and $x=4$;
and so on.

Example 8.

In what ratio must tea costing Rs 4·80 a lb. be mixed with tea costing Rs 3·80 a lb., so that the mixture can be sold for Rs 4·40 a lb. at a profit of 10%?

First find the cost price of the mixture.

The cost price of the mixture $=\frac{100}{110}$ of Rs 4·40 = Rs 4·00.

Now proceed as in Example 4.

The required rato is 1 : 4.

MISCELLANEOUS

Example 9.

Of a consignment of 26 doz. felt hats, some are sold at Rs 7·50 each and the rest at Rs 10·75 each. The average price worked out at Rs 9 each. How many were sold at the lower rate?

We first arrive at the proportions (as in example 4).

The ratio is 7 : 6.

The number of hats sold at the first price is, therefore,

$\frac{7}{13}$ of 26 doz., that is, $\underline{\text{14 doz.}}$ Ans.

Example 10.

In what proportions must two alloys of copper and tin, one containing 60% copper and the other 80% copper, be mixed, for the resulting alloy to contain 75% copper?

New alloy
75% copper.

alloy (1)
60%
15% more

alloy (2)
80%
5% less

The proportions are 5 parts of alloy (1) to 15 parts of alloy (2), or 1 : 3. Ans.

EXERCISES 50

1. In the following examples, find in what ratio the commodities of which the prices are given, must be mixed, in order that the cost price of the mixture may be as stated.

		Price of Mixture.
(*a*)	Tea at Rs 2 a lb. Tea at Rs 1·50 a lb.	Rs 1·75 a lb.
(*b*)	Coffee at Rs 5·00 a kg Coffee at Rs 3·50 a kg	Rs 4 a kg
(*c*)	Spirit at Rs 7·50 a l Water costing nothing	Rs 6·75 a l
(*d*)	Milk at Re 1 a l Water costing nothing.	Rs 0·75 a l
(*e*)	Coffee at Rs 2·25 a kg Coffee at Rs 1·75 kg Chicory at Rs 0·75 a kg	Rs 2 a kg.

2. In what ratio did a man mix two kinds of tea which cost him 2*s*. 6*d*. and 1*s*. 10*d*. a lb. respectively, if he was able to make a profit of 40% by selling the mixture at 2*s*. 10*d*. a lb.?

3. In what ratio must a liquid containing 88% spirit be mixed with a liquid containing 70% spirit for the mixture to contain 77% spirit?

4. How many kg of tea worth 3*s*. a kg must be mixed with 10 kg of tea costing 5*s*. a kg, in order that a profit of $33\frac{1}{3}$% may be made by selling the mixture at 5*s*. 4*d*. a kg?

5. By mixing two kinds of confectionery which are being sold at Rs 2·25 and Rs 3·00 a lb. respectively, a confectioner was able to sell the mixture at Rs 2·50 a lb. How would he have had to price the mixture had he reversed the proportions in which he mixed the confectionery?

6. A dealer buys 120 doz. apples at Rs 2·40 a doz. By selling some of them at the rate of Rs 2 a doz. and the rest at Rs 2·60 a doz., he just recovers his cost price from the combined transactions. How many apples did he sell at the higher rate?

7. A dealer buys 65 doz. apples at Rs 2 a doz. By selling some of them at the rate of Rs 1·75 a doz. and the rest at Rs 2·40 a doz., he makes a profit of 10%. How many apples did he sell at the higher price?

8. A dealer buys 40 doz. apples at Rs 2·25 a doz. Some of them go rotten and have to be thrown away. By selling the rest at Rs 3 a doz., he just recovers what he paid. How many apples had he to throw away?

9. In what ratio must a mixture of 70% spirit and 30% water be further diluted with water so as to drop the percentage of spirit by 10?

10. The following figures give the percentages of sand and clay in two different kinds of soil:—

(1)	Sand 70%	Clay 20%.
(2)	Sand 30%	Clay 65%.

In what proportions must the soils be mixed to obtain a soil containing

(*a*) 60% sand? What percentage of clay will this soil contain?

(*b*) 45% clay? What percentage of sand will this soil contain?

(*c*) equal quantities of sand and clay?

11. The following figures give the percentages of sand, clay and humus present in two different soils: (the remaining ingredients being considered negligible).

	Sand	Clay	Humus
(1)	80	15	5
(2)	20	55	25

In what ratio must the soils be mixed to obtain a soil containing:

(*a*) 30% clay? What will be the ratio of sand to humus in this soil?

(*b*) 60% sand? What percentage of humus will there be in this soil?

(*c*) equal quantities of sand and clay? What percentage of humus will this soil contain?

(*d*) 20% humus? What ratio of sand to clay will this soil contain?

Give the percentage composition of this soil.

12. The following figures give the percentages of sand and clay in three different soils:

	Sand	Clay
(1)	70	20
(2)	40	50
(3)	50	45

In what ratio must they be mixed

(*a*) to obtain a soil with 50% sand and 43% clay ?

(*b*) to obtain a soil with 45 % clay and 48% sand ?

CHAPTER XXII

CONVERSION

Density

Example 1.

A silver coin is 1·5 cm in diameter and 0·75 mm thick. How many such coins can be made from a bar of silver 21 cm long and of square cross-section of side 2·5 cm? (Take $\pi = 3{\cdot}1416$.)

The volume of one silver bar $= 21 \times 2{\cdot}5^2$ c.c. (i)

The volume of one coin $= \pi r^2 h$

$= 3{\cdot}1416 \times (0{\cdot}75)^2 \times 0{\cdot}075$ c.c. (ii)

The number of coins $= \dfrac{\text{(i)}}{\text{(ii)}}$

$$= \frac{21 \times 2{\cdot}5 \times 2{\cdot}5 \times 1000}{75 \times 0{\cdot}75 \times 0{\cdot}75 \times 3{\cdot}1416}$$

$= \underline{990}$. Ans.

Example 2.

A solid metal sphere of radius 6·3 in. is melted down and recast as a right-circular cylinder with a base of radius 4·5 in. Find the height of the cylinder.

$$\begin{aligned} \text{Volume of sphere} &= \tfrac{4}{3}\pi r^3 \\ &= \tfrac{4}{3}\pi 6{\cdot}3^3 \text{ cubic in.} \\ \text{Volume of cylinder} &= \pi r^2 h \\ &= \pi 4{\cdot}5^2 h \text{ cubic in.} \\ (\text{where } h &= \text{height of cylinder}). \\ \text{Then since } \pi 4{\cdot}5^2 h &= \tfrac{4}{3}\pi 6{\cdot}3^3 \\ h &= \tfrac{4}{3}\times\frac{6{\cdot}3^3}{4{\cdot}5^2} \\ &= \underline{16\tfrac{1}{2}} \text{ in. Ans.} \end{aligned}$$

EXERCISES 51

$\pi=\frac{22}{7}$. $\log \pi=0{\cdot}4971$.

1 c. c. of water weighs 1 gm.

1 c.c. of aluminium weighs 2·6 gm. 1 c.c. of lead weighs 11·4 gm.
1 c.c. of brass weighs 8·4 gm. 1 c.c. of iron weighs 7·2 gm.

Weight of
Aluminium: 162 lb. per cubic foot.
brass: 524 lb. per cubic foot.
lead: 711 lb. per cubic foot.
iron: 449 lb. per cubic foot.
water: 62·4 lb. per cubic foot.

Volume of sphere $=\frac{4}{3}\pi r^3$ cubic units.

Answers to the following examples are required to 3 significant figures.

1. Find the weight in gm of the following:—
   (*a*) a brass bar of length 15 cm and of uniform square cross-section of side 6 cm.
   (*b*) an iron bar of length 15 in. and of uniform square cross-section of side 3 in.
   (*c*) a flat circular ring of aluminium 0·25 cm thick, the internal diameter being 6 cm, and external diameter 9 cm.
2. Find the weight in gm of:—
   (*a*) a brass sphere of radius 5·3 cm.
   (*b*) a leaden sphere of radius 6·7 cm.
   (*c*) a cube of iron, side 5·2 cm.
   (*d*) a cylindrical iron rod of length 15 cm and of cross-section of radius 3·1 cm.

(*e*) a solid conical piece of aluminium of 7·2 cm radius and height 5·8 cm.

3. Find the weight in lb. of:—

(*a*) a bar of iron of length 15 in. and of square cross-section of side 2·1 in.

(*b*) a brass ball of radius 3·5 in.

(*c*) a conical piece of lead of radius 5·6 in. and height 6·3 in.

(*d*) a cylindrical rod of iron of radius 1·2 in. and length 21 in.

(*e*) a cube of aluminium of total surface 37·5 sq. in.

(*f*) a flat circular ring of aluminium 0·25 in. thick and of internal and external diameters 6 in. and 9 in. respectively.

4. Find the weight in gm of:—

(*a*) a hollow brass pipe of length 35 cm external diameter 5·3 cm; thickness of brass 4 mm.

(*b*) a hollow sphere of iron of external diameter 10·7 cm, and thickness 1·3 cm.

(*c*) a mug, cylindrical in shape, of height 12·5 cm, internal radius 6 cm made out of aluminium of uniform thickness 2 mm.

(*d*) a rectangular lidless box made out of brass sheeting 3 mm thick, the external dimensions being 19 cm by 15 cm by 10·4 cm (height).

5. Find the weight in lb. of:—

(*a*) a hollow iron sphere of external diameter 5·2 in. and thickness 1·3 in.

(*b*) a cylindrical mug of height 9 in. and external radius $4\frac{1}{2}$ in. made out of aluminium 0·05 in. thick.

(*c*) a lead pipe of external diameter 3 in., internal diameter 2·7 in. and length 15 ft.

(*d*) a brass pipe of the same dimensions as (*c*).

6. An iron dumb-bell is composed of equal spheres of radius 4 in. connected by a cylindrical bar of diameter 2 in. and length 6 in. What is the weight of the dumb-bell?

7. If 250 gm of glycerine are poured into a cylindrical bottle of internal dimeter 10 cm, to what height will it fill the bottle? (1 c.c. of glycerine weighs 1·3 gm.)

8. A cone of height 6 cm and with a base of diameter 8 cm is made of brass. How much heavier is it than an aluminium cone of the same diamensions?

9. A cylinder of iron is to be made so as to weigh 70 kg. If the radius of the base is to be 10·5 cm, how high must the cylinder be?

10. A brass bar of square cross-section is to be made so as to weigh 25 lb. If the length of the bar is to be 2 ft, what must the side of the cross-section measure?

11. A cylindrical stone bottle of internal diameter 4 cm and weight 250 gm is filled with mercury to a depth of 5 cm. What is the weight of the whole? (Assume that mercury weighs 13·6 gm per c.c.)

12. An aluminium cylinder, open at both ends, is 14 cm long and has an internal diameter of 10·6 cm. If it weighs 122·4 gm, find the thickness of the aluminium to the nearest $\frac{1}{10}$ mm.

13. A brass container of square cross-section of side 5·7 in. is filled with lead to a depth of 4·3 in. What is the weight of the lead?

14. A cylindrical hole of diameter 2 cm and depth 5·6 cm is drilled in a bar of iron 16 cm long and of uniform rectangular cross-section 9 cm by 7·5 cm. What will be the weight of the bar after the hole has been filled up with lead?

15. Through the entire length of a solid brass cylinder 6 in. long and of diameter 2 in. is drilled a hole of 1 in. diameter. What will be the weight of the cylinder after the hole has been filled up with lead?

16. In a cylindrical vessel of external depth 15 cm and internal diameter 12 cm, made out of aluminium 3 mm thick, is placed an iron ball of diameter 5 cm, and water is then filled up to the top. What is the weight of the whole? ($\pi=\frac{22}{7}$.)

17. A silver coin is 1·75 cm in diameter and 1 mm thick. How many such coins can be made from a bar of silver 25 cm long and of square cross-section of area 2·8 sq. cm?

18. A silver coin is 1·6 cm in diameter and 0·9 mm thick. How many such coins can be made from a cylindrical bar of silver 11 cm long and 3·15 cm in diameter?

19. A rectangular block of metal 43·55 cm by 28·36 cm by 9·75 cm is drawn into wire of a diameter 0·24 cm. Find in kilometres (to 2 significant figures) the length of the wire.

20. A cylindrical rod of metal 15 in. long and $3\frac{1}{2}$ in. in diameter is drawn into wire of diameter 0·05 in. Find the length of the wire in miles to 3 significant figures.

21. A floor $12' \times 20'$ is coated uniformly with paint. If 5 gallons of paint are used, what is the thickness of the coat?

22. How many brass spheres of diameter 5 cm will weigh the same as 55 cubes of iron 5 cm side? (Brass weighs 8·4 gm per c.c. and iron 7·2 gm per c.c.)

23. A brass block of weight 9 kg is drawn into wire of diameter 0·18 cm. What length of wire is obtained? (Answer in km to 2 significant figures.)

24. An open cylinder of external diameter 3·2 in. is cased with brass 0·3 in. thick. If 66 cubic inches of brass are required, what is the length of the cylinder?

25. A solid metal sphere of radius 7·5 in. is melted down and recast as a right-circular cylinder with a base of radius 2·4 in. Find the height of the cylinder.

26. A solid metal sphere of radius 8·6 in. is melted down and recast as a right-circular cylinder of height 3·5 in. Find the radius of the base of the cylinder.

27. A hollow cylinder (without ends) made of metal 0·4 in. thick is 2 feet long and of internal diameter 10 in. It is melted down and recast as a solid cylinder of height 6 in. What is the radius of the base of the solid cylinder?

28. A hollow sphere of metal of internal diameter 6 cm and external diameter 8·4 cm is melted down and recast as a rectangular bar of square cross-section. If the side of the cross-section is 2·1 cm how long is the bar?

29. A hollow sphere of metal of internal diameter 5·6 cm and external diameter 7·2 cm is melted down and recast as a cone of height 5 cm. What is the radius of the base of the cone?

30. A flat metal ring of internal diameter 4 cm and external diameter 5·2 cm and 0·5 cm thickness is melted down and recast as a cylindrical bolt of 0·5 cm diameter. How long is the bolt?

CHAPTER XXIII

AVERAGES

Example 1.

A cyclist cycles 10 miles at 8 miles an hour and then 5 miles at 6 miles an hour. What is his average speed for the whole journey ?

The time taken for the first 10 miles is $= \frac{10}{8}$ hrs. $= 1\frac{1}{4}$ hrs.

The time taken for the rest of the journey $= \frac{5}{6}$ hr.

The total time taken $= 1\frac{1}{4} + \frac{5}{6}$ hrs.

$= 2\frac{1}{12}$ hrs.

The total distance cycled $= 10 + 5 = 15$ miles.

$\therefore$ the average speed $= \dfrac{\text{total distance}}{\text{total time}}$

$= \dfrac{15}{2\frac{1}{12}}$ miles per hour

$= \dfrac{15 \times 12}{25}$

$= \underline{7\frac{1}{5}\text{ m.p.h.}}$ Ans.

Example 2.

A class is divided into two sections of 20 pupils and 30 pupils. The average marks in History of the first section was 60, and those of the second 55. What was the average for the whole class?

The total for section 1 was $60 \times 20 = 1200$ marks.

The total for section 2 was $55 \times 30 = 1650$ marks.

The total for the whole class was $1200 + 1650 = 2850$ marks.

The total number of pupils was $20 + 30 = 50$.

$\therefore$ the average marks were $\dfrac{2850}{50}$

$= \underline{57\text{ marks.}}$ Ans.

Example 3.

Five pints of one kind of fluid, weighing $1\frac{1}{2}$ lb. to the pint, are mixed with four pints of another kind weighing $1\frac{1}{4}$ lb. to the pint. What is the weight of a pint of the mixture?

The wt of 5 pt of 1st fluid $= 5 \times 1\frac{1}{2} = 7\frac{1}{2}$ lb.
The wt of 4 pt of 2nd fluid $= 4 \times 1\frac{1}{4} = 5$ lb.
$\therefore$ the wt of 9 pt of the mixture $= 12\frac{1}{2}$ lb.
$\therefore$ 1 pt of the mixture weighs $\dfrac{12\frac{1}{2}}{9}$ lb.
that is, $1\frac{7}{18}$ lb. Ans.

EXERCISES 52

1. The following are the heights of a sampling of trees of the same kind growing in a certain area:

40·7; 38·3; 43·6; 45·1; 35·6; 33·7; 42·4; 49·6; 38·2; 41·5 ft.

By how many feet does the height of the tallest tree exceed the average?

2. The following are the rainfall figures for July 1936 in a certain district:

July 1st, 7th, 19th, 27th (on each day)	... 1·30″
July 4th	... 5·75″
July 11th	... 2·60″
July 12th	... 3·54″
July 16th, 25th	... 1·10″
July 2nd, 3rd, 31st	... 2·00″
The other days	... nil

Find the average daily rainfall for the month (correct to the first decimal place).

3. A class is divided up into two sections of 16 pupils and 15 pupils. The average marks in a certain subject are 53 for the first section, and 60 for the second. What is the average for the whole class?

4. There are five balls in a bag. Two of them weigh $4\frac{3}{4}$ oz. each, and the other three weigh $4\frac{1}{4}$ oz. each. If they are to be replaced by five balls of equal weight, what should each of these five weigh provided that the total weight of the bag is to be the same?

5. What is the average of the following fractions:

$\frac{3}{4}$; $\frac{5}{6}$; $\frac{1}{2}$; $\frac{3}{8}$?

6. In a class of 30 pupils, two were awarded 73 marks, one received 70 marks, five received 62 marks, three received 60 marks, four received 57 marks, six received 50 marks. The next five received 40 marks, and the remainder 20 each. What were the average marks of the class? How much higher would the average have been if in the last batch of pupils, the ones that got 20 had been awarded 10 more marks each?

7. Two boys' scores at a shooting range were

1st boy : 3 bulls; 5 inners; 2 magpies; 4 outers; 1 miss.

2nd boy : 5 bulls; 7 inners; 0 magpies; 5 outers; 3 misses.

Which boy had the better average?

(A bull scores 4 points; an inner, magpie and outer, 3,2,1 respectively.)

8. Three qualities of coffee are mixed together as follows:

five kg. @ 1*s.* $4\frac{1}{2}$*d.* a kg
seven kg. @ 1*s.* 6*d.* a kg
three kg. @ 1*s.* $8\frac{1}{2}$*d.* a kg.

What is the average price of the mixture?

9. The following are the details of the running of an express train between two stations X and Y.

Leaves X at 9.40 a.m., arrives at A 10.00 a.m.
,, A at 10.10 a.m., ,, at B 11.05 a.m.
,, B at 11.15 a.m., ,, at C 11.45 a.m.
,, C at 11.50 a.m., ,, at Y 1.00 p.m.

The distances XA, AB, BC, CY are respectively 16 km, 43 km, 25 km and 56 km.

Find (i) the average speed between X and A.

(ii) the average speed for the whole run, neglecting halting time.

10. A cyclist cycles 8 miles at 12 miles an hour; and then 15 miles at 9 miles an hour; and then 7 miles at 8 miles per hour. What is his average speed?

11. A train runs for $\frac{1}{4}$ hour at 50 miles an hour and then $\frac{1}{2}$ hour at 45 miles an hour. What is the average speed for the whole journey ?

12. A motorist motors to a place 100 km away and back. He goes at 20 km an hour and returns at 30 km an hour. What is his average speed ?

13. What would be the answer to No. 12, if the motorist went at 30 km an hour and returned at 20 km an hour ?

14. The average of five fractions is $\frac{1}{2}$. Four of them are $\frac{3}{8}$, $\frac{3}{5}$, $\frac{3}{4}$, $\frac{2}{5}$. What is the fifth?

15. A boy's batting average for seven innings is 15 runs. If he scores 31 in his next innings, by how much will he increase his average?

16. A boy's batting average for ten innings is 12. What should he score in his next innings to increase his average to 15 ?

17. A class is divided up into two sections of 16 and 15 boys respectively. Their respective average marks in a certain subject were found to be 51 and 47. It was found later, however, that a mistake had been made. B. Jones, who scored 61 and belonged to the first section, was included in the second section; while C. Jones, who scored 73 and belonged to the second section, was counted in the first section. Give the correct average for the two sections.

18. The average of the six numbers 15, 42, *x*4, 61, 28 and 30 is 3*. What digits do the *x* and the * stand for ? (Do not take *x* as an algebraical symbol, but just an ordinary digit.)

19. The sum of five consecutive numbers is 70. What are they ?

20. The sum of six consecutive numbers is 153. What are they ?

21. The average yearly rainfall of a certain place, calculated on the figures for 1954, 1955 and 1956, is 65 inches. Calculated on the figures for 1953, 1954 and 1955 the average is 63 inches. The actual rainfall for 1956 was 60 inches. What was the rainfall for 1953 ?

CHAPTER XXIV

TRIGONOMETRY

Ratios: (Sine, Cosine, Tangent)

Angle BAO$=90°$.

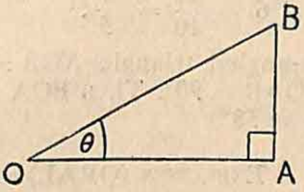


Fig. 24

Let angle AOB be called θ (Theta).

Let AB be called the opposite side since it is opposite the angle θ.

Let AO be called the adjacent side because it is adjacent to the angle θ.

OB is called the hypotenuse, for it is the side opposite the right angle.

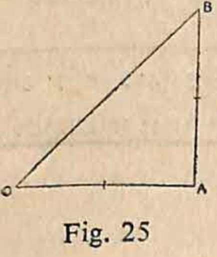
$$\text{Sine } \theta = \frac{AB}{BO} = \frac{\text{Opposite side}}{\text{Hypotenuse}}.$$

$$\text{Cosine } \theta = \frac{AO}{OB} = \frac{\text{Adjacent side}}{\text{Hypotenuse}}.$$

$$\text{Tangent } \theta = \frac{AB}{OA} = \frac{\text{Opposite side}}{\text{Hypotenuse}}.$$

Sin. is the short for sine.
Cos. is the short for cosine.
Tan. is the short for tangent.

Example 1. Find tan 45° by a geometrical construction.



Make angle AOB$=45°$.

Let OA $=$ 2 in.

Draw AB perpendicular to OA. Angle ABO $= 45°$.

$\therefore$ AB $=$ 2 in.

$\therefore \tan 45° = 1$.

Fig. 25

Example 2. Find tan 56°, tan 60°, tan 30°, using a geometrical construction. (Hint: Make angle AOB = 56° and OA = 5 cm and then complete the right-angled triangle.)

Example 3. Find by construction an angle of which the tangent is 1·6.

$$1{\cdot}6 = \frac{16}{10} = \frac{8}{5}.$$

Construct a right-angled triangle AOB so that OA = 5 cm, AB = 8 cm, angle OAB = 90°. Then BOA is the required angle. It will be found to be 58°.

EXERCISES (ORAL)

In the following diagrams angle A is a right angle. State the tangents of the angles marked C.

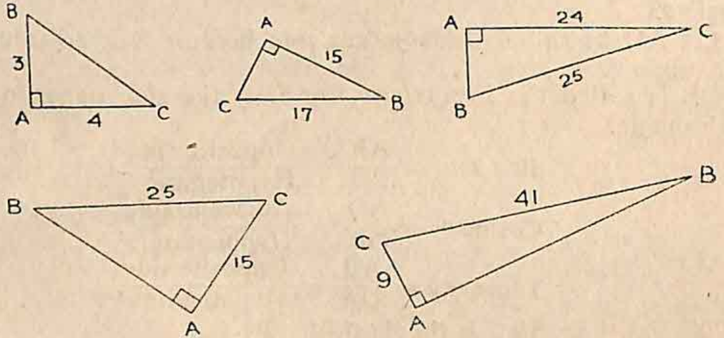


Fig. 26

Example 4. Use tables of natural tangents to find tan 26°36′.

Difference

·	0′	10′	20′	30′	40′	50′	1′	2′	3′	4′	5′	6′	7′	8′	9′
26°	0·4877	0·4913	0·4950	0·4986	0·5022	0·5059	4	7	11	15	18	22	25	29	33

Tan 26° 30′ is 0·4986.

Difference for 6′ is ·0022.

∴ tan 26° 30′ by adding is 0·5008.

EXERCISES (Oral)

Use tables to read the tangents of the following angles:

(*a*) 20°; 20° 10′; 20° 15′; 20° 57′.
(*b*) 15° 15′; 15° 32′; 15° 59′.
(*c*) 25° 25′; 37° 13′; 45° 60′; 72° 15′.
(*d*) 61° 19′; 71° 15′; 75°; 57° 57′.

Example 5. The angle of elevation of a tower seen from a distance of 400 feet is 32°. Find the height of the top of the tower above the point of observation.

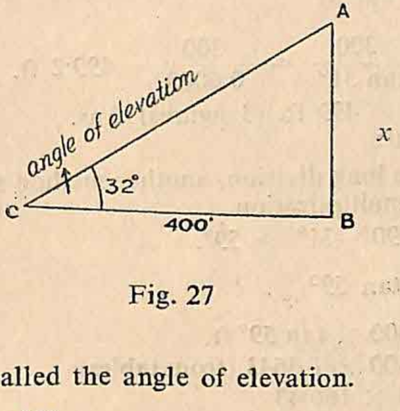


Fig. 27

Angle ACB is called the angle of elevation.

$$\frac{x}{400} = \tan 32°.$$

$$\begin{aligned} \therefore x &= 400 \tan 32° \text{ ft.} \\ &= 400 \times 0{\cdot}6249 \text{ (from tables)} \\ &= 4 \times 62{\cdot}49 \\ &= 249{\cdot}96 \text{ ft.} \end{aligned}$$

249·96 ft. or 250 ft. to 3 figures. Ans.

Example 6.

The angle of depression of a boat seen from the top of a vertical cliff 300 ft. high is 31°. Calculate the distance of the boat from the foot of the cliff.

First Method:

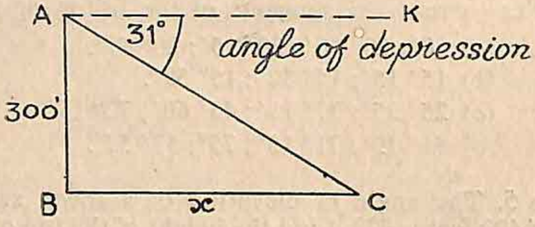


Fig. 28

since KAC = 31° ∴ ACB = 31° (alternate angle).

$$\therefore \frac{300}{x} = \tan 31°$$

$$\text{or } x = \frac{300}{\tan 31°} = \frac{300}{0{\cdot}6009} = 499{\cdot}2 \text{ ft.}$$

499 ft. (3 figures) Ans.

Second Method:

To avoid the long division, another method may be used which involves only multiplication

$\angle$ CAB = 90°−31° = 59°.

$$\text{Then } \frac{x}{300} = \tan 59°$$

$$\begin{aligned} \text{or } x &= 300 \times \tan 59° \text{ ft.} \\ &= 300 \times 1{\cdot}6643 \text{ (from tables)} \\ &= 3 \times 166{\cdot}43 \\ &= 499{\cdot}29 \text{ ft.} \end{aligned}$$

∴ The distance of the boat from the foot of the cliff = 499 ft. (3 figures) Ans.

EXERCISES 53

Use the tangent ratio for Nos. 1—5.

1. The angle of depression of an object seen from a church spire is 32°. If the object is 400 feet from the foot of the spire find the height of the spire.

2. A vertical pole is 10 metres long, and the angle of elevation

of the top of the pole as seen from a point X is 35°. Find the distance of X from the foot of the pole.

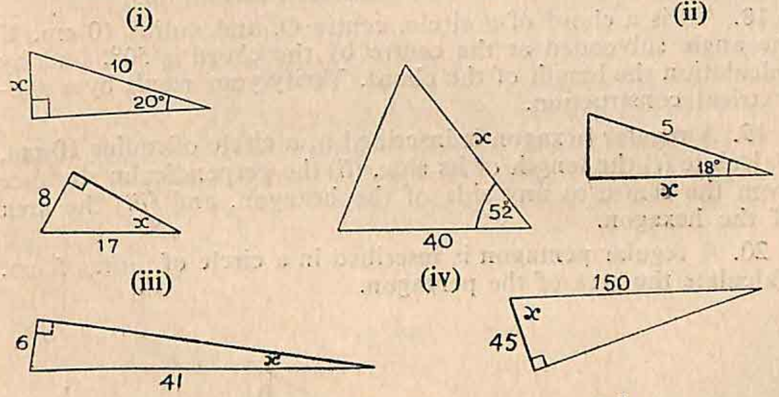
3. ABC is an isosceles triangle, and AD is the perpendicular to BC; find the length of AD if angle B=37° and BC=6 in. If angle A is 96° and AD=5 in., find the length of BC.

4. A stick leaning against a wall makes an angle of 21° with the wall; the foot of the wall is 0·6 metres from the wall. How high does the stick reach up the wall?

5. ABC is an equilateral triangle. AD is perpendicular to BC. If the side of the triangle is 2 in. find the value of tan 60° and tan 30°. Use the sine and cosine ratios.

6. Find by means of a geometrical construction the value of (*i*) sin 30° (2) sin 60° (3) cos 30° (4) cos 60°.

7. Find the value of x in the following triangles:



(v) Fig. 29 (vi)

8. Use your tables to write down the (a) sine (b) cosine of each of the following angles:

62°; 35°; 48°; 57°; 76°; 85°; 16°; 27°; 45′; 81° 17′; 72° 14′; 55° 55′; 36° 37′; 48° 49′; 31° 39′; 25° 59′.

9. If sin A=3/5 and A is acute, find the value of (a) tan A (b) cos A.

10. If sin A=8/17 and A is acute, find the values of cos A and tan A.

11. If sin A = 3/5, find the value of $\cos^2 A+\sin^2 A$.

12. If cos A = $\frac{9}{41}$, find the value of $\cos^2 A + \sin^2 A$.

13. If $\cos\theta = \frac{m^2-n^2}{m^2+n^2}$, find sin θ and tan θ.

14. If an isosceles △ XYZ has its base YZ 3 cm long and the vertical angle 70°, find the length of the equal sides.

15. The legs of a pair of dividers are 6 cm long. To what angle must they be opened so that the ends are 4 cm apart?

16. X, Y, W, Z are the corners of a trapezium. XY is parallel to WZ. The diagonal YZ = 60, angle XZY=90°, angle ZXY=50°, angle ZYW = 70°. Calculate the length of XY, the perpendicular distance between the parallels and the length of YW to the first decimal place.

17. The angle of elevation of a lighthouse is 18°, 109 metres nearer it is 40°. Find the height of the lighthouse and the distance of its base from the first point of observation.

18. AB is a chord of a circle, centre O, and radius 10 cm. If the angle subtended at the centre by the chord is 50°, find by calculation the length of the chord. Verify your result by a geometrical construction.

19. A regular hexagon is inscribed in a circle of radius 10 cm. Calculate (*i*) the length of its side, (*ii*) the perpendicular distance from the centre to any side of the hexagon, and (*iii*) the area of the hexagon.

20. A regular pentagon is inscribed in a circle of radius 5 cm. Calculate the area of the pentagon.

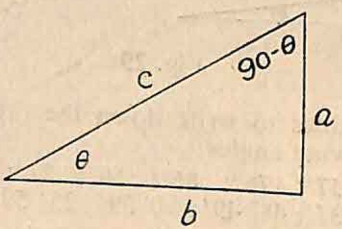


Fig. 30

$\sin\theta = \frac{a}{c}$;

$\cos(90°-\theta) = \frac{a}{c}$; $\therefore \sin\theta = \cos(90°-\theta)$.

Sin $(90° - \theta) = \frac{b}{c}$;

cos $\theta = \frac{b}{c}$; $\therefore$ cos θ = sin $(90° - \theta)$.

Sin $\theta = \frac{a}{c}$;

cos $\theta = \frac{b}{c}$; $\therefore \frac{\sin\ \theta}{\cos\ \theta} = \frac{a}{c} \div \frac{b}{c} = \frac{a}{c} \times \frac{c}{b}$

$$= \frac{a}{b} = \tan\ \theta.$$

$$\therefore \frac{\cos\ \theta}{\sin\ \theta} = \frac{1}{\tan\ \theta} = \cot\ \theta.$$

Cotangent $\theta = \frac{b}{a}$. Secant $\theta = \frac{c}{b}$.

Cosecant $\theta = \frac{c}{a}$.

Sin $\theta = \frac{a}{c}$; $\therefore \sin^2\ \theta = \frac{a^2}{c^2}$. (1)

Cos $\theta = \frac{b}{c}$; $\therefore \cos^2\ \theta = \frac{b^2}{c^2}$. (2)

Adding (1) and (2) we get

$$\sin^2\theta + \cos^2\theta = \frac{a^2}{c^2} + \frac{b^2}{c^2}$$

$$= \frac{a^2+b^2}{c^2}$$

$= \frac{c^2}{c^2}$ (Pythagoras' Theorem)

$$= 1.$$

Trigonometrical Ratios of 30°, 60°, 45°.

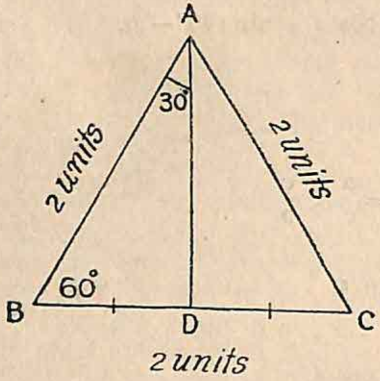


Fig. 31

ABC is an equilateral triangle.

AD is perpendicular to BC.

Then BC is bisected at D.

BD = 1 unit.

In △ ABD

$$AD^2 = 2^2 - 1^2 = 4 - 1 = 3.$$

$$\therefore AD = \sqrt{3}.$$

$$\text{Now Sin } 30^\circ = \frac{BD}{AB} = \frac{1}{2}.$$

$$\text{Sin } 60^\circ = \frac{AD}{AB} = \frac{\sqrt{3}}{2}.$$

$$\text{Cos } 30^\circ = \frac{AD}{AB} = \frac{\sqrt{3}}{2}.$$

$$\text{Cos } 60^\circ = \frac{BD}{AB} = \frac{1}{2}.$$

$$\text{Tan } 30^\circ = \frac{BD}{AD} = \frac{1}{\sqrt{3}}.$$

$$\text{Tan } 60^\circ = \frac{AD}{BD} = \sqrt{3}.$$

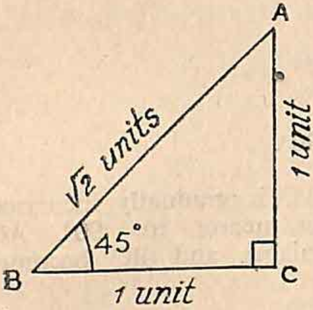


Fig. 32

ABC is an isosceles triangle right-angled at C.

AC = BC = 1 unit.

$AB^2 = 1^2 + 1^2 = 2.$ $\therefore AB = \sqrt{2}.$

$\angle ABC + \angle BAC = 90°.$

$\therefore \angle B = \angle A = 45°.$

$$\text{Now, } \sin 45° = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\text{and } \tan 45° = 1.$$

Angle of 0°.

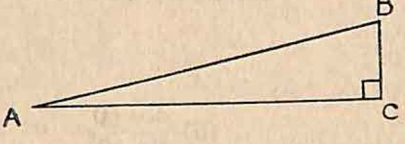


Fig. 33

If angle BAC is very small, BC is very small in comparison with AB, and AC is nearly equal to AB. Now as angle BAC gets gradually smaller, the length of BC decreases and ultimately becomes zero and AC becomes equal to AB.

Hence sin 0° = 0, cos 0° = 1 and tan 0° = 0.

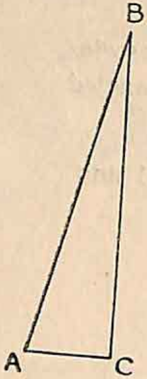


Fig. 34

Angle of 90°.

If angle BAC is gradually increased so as to get nearer to 90°, AC ultimately vanishes and BC becomes equal to AB.

$\therefore \sin 90^\circ = 1$

and $\cos 90^\circ = 0$,

and $\tan 90^\circ = \infty$ (infinity).

This means that as the angle gets nearer to 90° the tangent of an angle increases without limit.

EXERCISES 54

Find the value of:

(*i*) $\frac{\sin 30^\circ}{\cos 30^\circ}$

(*ii*) $\frac{\cos 60^\circ}{\sin 60^\circ}$

(*iii*) $\frac{\sin 30^\circ}{\tan 45^\circ}$

(*iv*) $\frac{\tan 60^\circ}{\cos 30^\circ}$

(*v*) $\sin 30^\circ \cos 30^\circ$

(*vi*) $\sin 60^\circ \cos 60^\circ$

(*vii*) $\sin 90^\circ + 6 \sin 30^\circ$

(*viii*) $\sin^2 30^\circ + \cos^2 30^\circ$

(*ix*) $\sin^2 45^\circ + \cos^2 45^\circ$

(*x*) $\sin^2 30^\circ + \cos^2 60^\circ + \sin^2 45^\circ$.

CHAPTER XXV

Three-Dimensional Problems

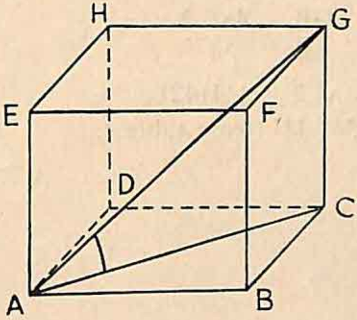


Fig. 35

Definition:

The projection of a straight line on a plane is the locus of the feet of the perpendiculars drawn to the plane from all points on the line.

In the above figure AC is the projection of the line AG.

The angle made by a straight line with a plane is the angle between the line and its projection on the plane.

Example: Find the angle which a diagonal of a cube of side 4 cm makes (a) with a face of the cube (b) with an edge of the cube.

(*a*) Required to find angle GAC.

$$AC^2 = AB^2+BC^2 = 16+16 = 32 \text{ cm};$$

$$\therefore\ AC = 4\sqrt{2}\text{ cm}.$$

$$\text{Tan GAC} = \frac{GC}{AC}=\frac{4}{4\sqrt{2}}=\frac{1}{\sqrt{2}}= \cdot 70711;$$

$$\therefore\ \angle GAC = 35^\circ\ 16' \text{ (from tables).}$$

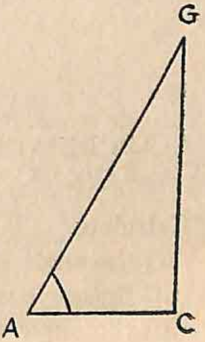


Fig. 36

(*b*) Required to find GAB.

$$\text{Tan GAB} = \frac{GB}{AB} = \frac{4\sqrt{2}}{4}$$

$$= \sqrt{2} = 1{\cdot}41421$$

or ∠ GAB = 54° 44′ (from tables).

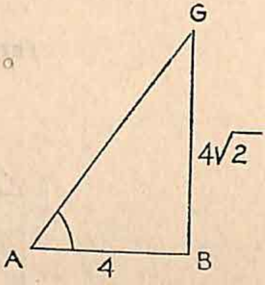


Fig. 37

Exercises 55

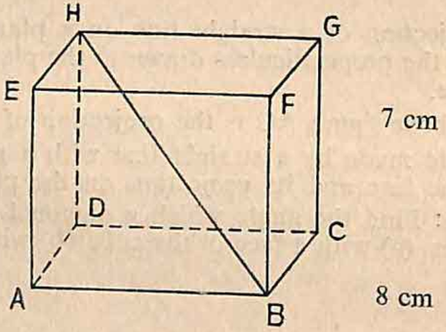


Fig. 38

1. ABCDEFGH is a rectangular block in which AB = 10 cm, BC = 8 cm, GC = 7 cm.

Calculate

(*i*) the angle HB makes with the plane ABCD.

(*ii*) the angle which the plane HAB makes with ABCD.

(*iii*) the angle HB makes with AB.

(*iv*) the angle HB makes with the plane ABFE.

2. A cardboard is folded as shown in the figure.

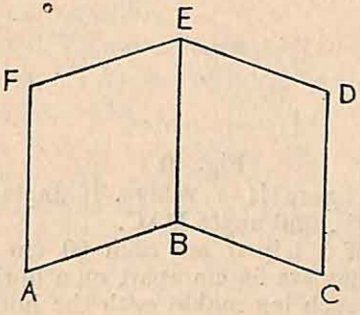


Fig. 39

If angle ABC is a right angle, find the angle AD makes with the plane ABC.

3. A cane 60 cm long stands in a corner of the room, the foot of the cane being 10 cm from one wall and 6 cm from the other. Find the inclination of the cane to the horizontal.

4. A pole stands at the corner P of a level square field PQRS each side of which is 100 metres long. If the elevation of the pole from Q is 15° 36′, find the height of the pole and its angle of elevation from R.

5. A flagstaff stands at the corner of a square playground. If it subtends an angle of 20° at the point where the diagonals of the field intersect, what angle does it subtend at each of the other three corners?

6. A pyramid stands on a square base of side 8 in. long. If the vertical height of the pyramid is 6 in., calculate the angle (*i*) an edge makes with the base, (*ii*) a slant face, with the base.

7. OABCD is a right pyramid standing on a square base ABCD, and O is its vertex. If AB = 5 cm and OA = 8 cm, find (*i*) the angle OD makes with ABCD, (*ii*) the angle between the planes OAD and OBC.

8. A hemispherical bowl of diameter 18 cm is suspended from a hook by three chains equidistant from one another. If each chain is 30 cm long, find the angle between two of the chains.

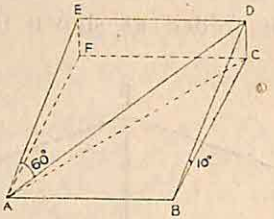


Fig. 40

9. The above figure is a wedge. If angle DBC = 10° and angle EAD = 60°, find angle DAC.

10. The legs of a tripod are each 60 cm long. If they are opened so that they are 36 cm apart on a horizontal plane, find the angle which each leg makes with the horizontal.

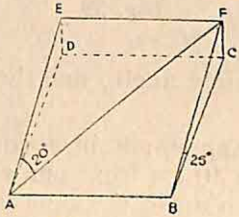


Fig. 41

11. In the above figure if angle FBC = 25°, angle EAF = 20°, and AF = 100 metres, find the length of CF.

In the given figure, ABCD is a rectangle, and TD is per-

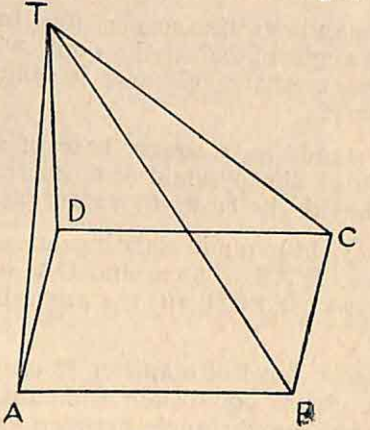


Fig. 42

pendicular to the plane ABCD.

If TD is 30 cm, AB is 50 cm, and BC 40 cm long, find

(*i*) the length of TB,
(*ii*) the angle TB makes with the plane ABCD,
(*iii*) the angle TC makes with the vertical,
and (*iv*) the length of TA.

13. The lid of a desk is a rectangle 24 in. by 18 in. The two longer sides are horizontal and one of them is 3 in. vertically above the other. Calculate the angles made with the horizontal by an 18 in. side and by a diagonal of the lid.

14. A right pyramid VABCD has a square base ABCD of side 5 in. and a vertex V at a height 4 in. above the centre of the base. Calculate

(*i*) the angle between the face VAB and the base ABCD,
(*ii*) the angle between the edge VA and the base ABCD
and (*iii*) the length of the edge VA.

15. A pyramid on a square base of side 10 cm is 12 cm in height. Calculate (*i*) its total surface area, (*ii*) the length of a slant edge and (*iii*) the inclination of a face to the base.

CHAPTER XXVI

The Sine Rule

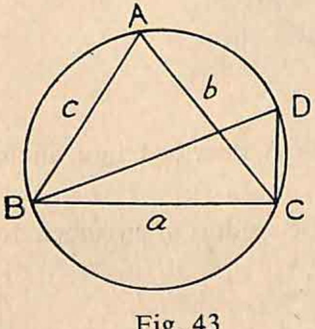


Fig. 43

To prove that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R,$$

R being the radius of the circumcircle.

ABC is an *acute*-angled triangle (p. 173) inscribed in a circle. BD is a diameter of the circle. CD is joined.

$\angle$BCD is 90°. (angle in a semi-circle)

$\angle$BDC = $\angle$BAC. (angles in the same segment)

In $\triangle$ BCD, $\frac{a}{\text{BD}} = \sin \text{BDC} = \sin \text{A}$.

$$\therefore \frac{a}{2\text{R}} = \sin \text{A};$$

$$\therefore \frac{a}{\sin \text{A}} = 2\text{R}.$$

Similarly it can be proved that

$$\frac{b}{\sin \text{B}} = 2R, \text{ and } \frac{c}{\sin \text{C}} = 2R.$$

$$\therefore \frac{a}{\sin \text{A}} = \frac{b}{\sin \text{B}} = \frac{c}{\sin \text{C}} = 2R.$$

Learn these :

$\sin(180° - \text{A}) = \sin \text{A}$.

$\cos(180° - \text{A}) = -\cos \text{A}$.

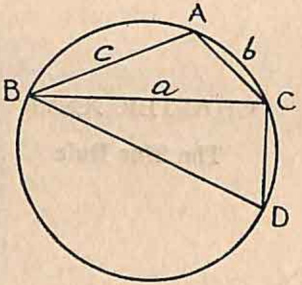


Fig. 44

ABC is a triangle with *obtuse* angle A inscribed in a circle. To prove that

$$\frac{a}{\sin \text{A}} = \frac{b}{\sin \text{B}} = \frac{c}{\sin \text{C}} = 2R. \ (R\text{=radius of circumcircle})$$

Draw BD the diameter, and join CD.

Then BCD = 90°.

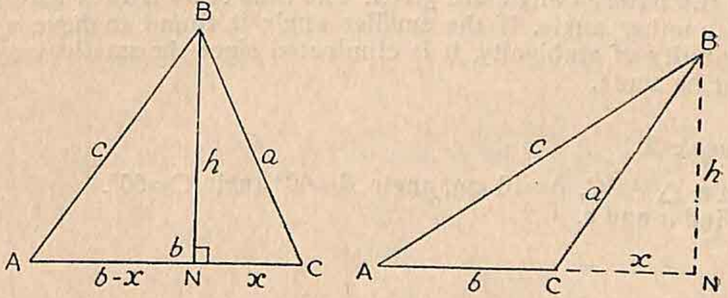
$\angle BDC = 180° - A$ ($\because$ in cyclic quadrilateral).

$$\text{In BCD} \quad \frac{a}{BD} = \sin BDC$$
$$= \sin(180° - A)$$
$$= \sin A.$$
$$\therefore \frac{a}{\sin A} = BD = 2R.$$

Similarly it can be proved that

$$\frac{b}{\sin B} = 2R \text{ and } \frac{c}{\sin C} = 2R.$$
$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = = 2R.$$

The Cosine Rule



(i) Fig. 45 (ii)

ABC is a triangle and BN $(=h)$ is perpendicular to AC. Let CN $=x$.

Since BNA is a right angled-triangle

$$BA^2 = AN^2 + BN^2$$

i.e. $c^2 = (b-x)^2 + h^2$ (Fig. i) or $c^2 = (b+x)^2 + h^2$ (Fig. ii)

$= b^2 + x^2 - 2bx + h^2$ $\qquad = b^2 + x^2 + 2bx + h^2$

$= b^2 + a^2 - 2bx$ $\qquad = b^2 + a^2 + 2bx$

since $h^2 + x^2 = a^2$ in right-angled ΔCNB.

But in Fig. (i) $x = a \cos C$ and in fig. (ii) $x = a \cos BCN$

$$= a\cos(180° - C)$$
$$= -a \cos C.$$

$\therefore c^2 = a^2+b^2-2ab \cos C$ in both figures.

This is the Cosine Rule.

Also since $c^2 = a^2+b^2-2ab \cos C$,

$$\therefore 2ab \text{ Cos } C = a^2+b^2-c^2;$$

$$\therefore \text{Cos } C = \frac{a^2+b^2-c^2}{2ab}.$$

Similarly

$$b^2 = c^2+a^2-2ac \cos \text{B and Cos B} = \frac{c^2+a^2-b^2}{2ac}$$

and $a^2 = b^2+c^2-2bc \cos \text{A}$ and $\text{Cos A} = \frac{b^2+c^2-a^2}{2bc}$.

Hence if the three sides of a triangle are known, the angle A, or angle B or angle C, as the case may be, can be calculated.

This Cosine Rule is used to find the third side where two sides and the *included* angle are given. The Sine Rule is then used to find another angle. If the smaller angle is found to have any possibility of ambiguity, it is eliminated since the smaller angle must be acute.

Example 1.

In a △ ABC, $b=10$ cm, angle $B=40°$ angle $C=60°$. Find a and c.

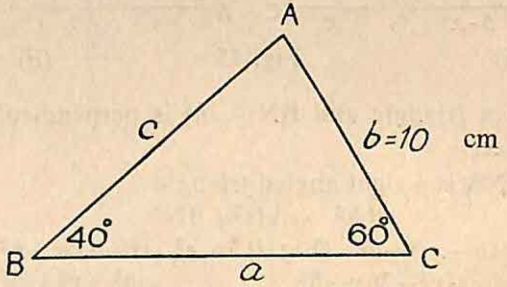


Fig. 46

By the Sine Rule $\frac{a}{\sin \text{A}} = \frac{b}{\sin \text{B}} = \frac{c}{\sin \text{C}}$.

But $\text{A} = 180° - 100° = 80°$.

$$\therefore \frac{a}{\sin 80^\circ} = \frac{10}{\sin 40^\circ}$$

$$\text{or } a = \frac{10 \times \sin 80^\circ}{\sin 40^\circ}$$

$$= 15{\cdot}31 \text{ cm.}$$

No.	log.
10	1·0000
sin 80°	$\bar{1}$·9934
	0·9934
sin 40°	$\bar{1}$·8081
15·31	1·1853

$$\frac{c}{\sin 60^\circ} = \frac{10}{\sin 40^\circ}$$

$$\therefore c = \frac{10 \sin 60^\circ}{\sin 40^\circ}$$

$$= \frac{10 \times {\cdot}866}{\sin 40^\circ} = \frac{8{\cdot}66}{\sin 40^\circ}$$

$$= \frac{8{\cdot}66}{\sin 40^\circ}$$

$$= 13{\cdot}47 \text{ cm.}$$

No.	log.
8·66	0·9375
sin 40°	$\bar{1}$·8081
13.47	1·01294

Example 2.

In a △ ABC $a=10$ cm, b=12 cm, angle $C=50^\circ$; find c.

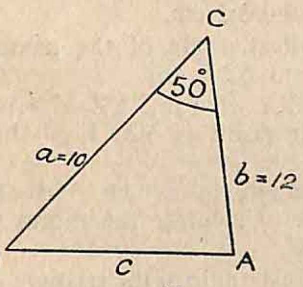


Fig. 47

By the Cosine Rule

$$\begin{aligned}
c^2 &= a^2+b^2-2ab \cos 50^\circ \\
&= 100+144-240 \cos 50^\circ \\
&= 244-240 \cos 50^\circ \\
&= 240-154{\cdot}3 \\
&= 85{\cdot}7 \\
\therefore c &= \sqrt{85{\cdot}7} = 9{\cdot}257 \\
\therefore c &= 9{\cdot}26 \text{ Ans.}
\end{aligned}$$

No.	log.
240	2·3802
cos 50°	$\bar{1}$·8081
154·3	2·1883

EXERCISES 56

Solve the following triangles as indicated:

1. $a=5$, $b=8$, $C=18°12'$; find c.

2. $a=10$ cm, $B=40°$, $C=60°$; find A, b.

3. $b=5''$, $c=8''$, $A=60°$; find a.

4. $c=8.6''$, $A=68°\ 20'$, $C=36°\ 10'$; find B,a,b.

5. $b=11$ cm, $c=8$ cm, $A=96°\ 19'$; find a.

6. $a=5'\ 4''$, $b=4'\ 2''$, $C=52°\ 6'$; find the remaining sides and angles.

7. $a=4$ cm, $b=5$ cm, $c=6$ cm; find the three angles.

8. The two hands of a clock are 8 in. and 6 in. long. What is the distance between the hands (1) at 4=o'clock (ii) half past three?

9. In a triangle ABC, BC=14, CA=15 and AB=13; find the values of Cos C, sin C and sin A, without using tables. Give your answers as fractions in their lowest terms.

10. ABCD is a parallelogram with adjacent sides 7 cm and 9 cm long. If the angle included by the two adjacent sides AB and AD is 110°, find the angle CAD.

11. Find the greatest angle of the triangle the sides of which are 2·1 cm, 2·6 cm, and 3·3 cm.

12. Find the smallest angle of the triangle whose sides are 77·7 cm, 65 cm, and 52·5 cm.

13. In a △ XYZ, XY=8·6 in., YZ=7·4 in. and angle XYZ= 34°. D is the middle point of YZ. Find the length of XD and the angle YXD.

14. ABC is a triangle in which AB=46 cm, AC=33 cm. and ∠ACB=47°12′. Calculate the radius of the circle passing through A, B and C.

15. Find the smallest angle of the triangle of which the sides are 15, 17, and 24 cm.

CHAPTER XXVII

COMPASS BEARINGS

All directions in a horizontal plane are described as follows: In the figure given the direction of P from O is obtained by facing North and then turning through 30° towards the East, it is described as N 30°E. Similarly the direction of Q is written as S 47°E.

All directions are measured either from North or from the South, NEVER from the east or the west. For example the direction of R is *not* W 25°N but is written as N 65°W. ∠RON =65°.

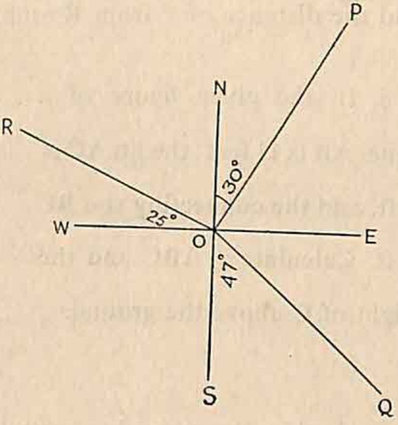


Fig. 48

There is another method of indicating direction. The angle which a given direction makes with the north line, measured in a clockwise direction is called the TRUE BEARING. The true bearing of P from O is 30°. The true bearing of Q from O is angle NOQ i.e. $90° + 43° = 133°$. The true bearing of R from O is reflex angle RON, i.e., $270° + 25° = 295°$.

EXERCISES 57

1. A lighthouse is 8 km due North of a harbour, and a boat sails from the harbour on a bearing of $67\frac{1}{2}°$ until the bearing of the lighthouse from the boat is 315°; the boat then sails towards the lighthouse till the bearing of the harbour from the boat is 225° and then returns to the harbour. Find the total distance sailed by the boat.

2. A man walking due E along a straight road observes a tree in a direction of 72°. After walking 1500 metres he observes that the bearing of the tree is 67°. How far is the tree from the road ?

3. Three pillars X, Y, Z are situated such that the distances XY, YZ are each 200 metres. The bearing of X and of Z from Y are 245° and 42° 30′. Find by calculation the bearing of Z from X and the distance between X and Z.

4. P, Q, R are three towns. Q is 10 km from P and on a bearing of 47°; R is 17 km from Q and on a bearing of 290°. Find the distance of P from R and the bearing of R from P.

5. In the given figure of a crane, AB is 11 feet; the jib AC is 22 ft, and the connecting rod BC 14 ft. Calculate ∠ABC and the height of C above the ground.

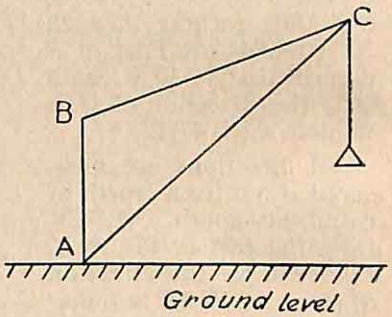


Fig. 49

6. A ship steaming due west is 3 nautical miles away in a direction N 30°W. Five minutes later her direction is N 50°W. What is her speed in knots ?

7. XYZ is a triangle such that angle X=37°, angle Y=67°. From a point P on XY produced a tangent is drawn to the circle passing through X, Y, Z and touching the circle at Z. If XY= 3·4 cm, calculate the length of PZ.

8. A is a fort 35 km due East of another fort M; another fort X is in a direction 160° from M and 230° from A. How far is X from M to the nearest kilometre ?

TEST PAPERS

Test 1

1. How many minutes will a man take to walk 4 miles, if he takes 84 steps each of 2 feet 9 inches in every minute? How many steps of the same length must he take each minute to cover the 4 miles in one hour?

2. (*a*) Simplify $(5\frac{2}{7}-3\frac{1}{3})\div(8\frac{1}{3}-4\frac{5}{6}-3\frac{2}{5})$.
(*b*) Find the cost of 537 books at £2 17*s* 10*d*. each.

3. Find the circumference and area of a circle of radius 13·3 cm.

4. By selling an article for Rs 66 a man gains $\frac{3}{8}$ of what it cost him. Find his gain per cent if he sells it for Rs 78.

5. At what rate per cent per annum will the interest on Rs 26100 amount to Rs 348 in 4 months?

6. X is 2 km from Y in the direction of 242°; Z is 3 km from X on a bearing 195°. Find the distance and bearing of Z from Y.

7. Prove that in an acute-angled △ ABC

$$a^2=b^2+c^2-2bc\cos A.$$

If $a=10$, $b=12$, $c=15$, calculate the angle ABC.

8. A tank which has a square base of side 2 feet contains water to a depth of 9 inches. An iron cylinder of radius 4 inches and length 15 inches is placed on the bottom of the tank with its axis horizontal. How much does the water level rise?

If the cylinder had been placed with its axis vertical, how much would the water level have risen? $\pi=3{\cdot}142$.

Test 2

1. Simplify $\dfrac{(\frac{3}{13}+\frac{4}{5})\times\frac{3}{11}}{\frac{3}{13}+\frac{4}{5}\times\frac{3}{11}}$.

2. By what per cent must Rs 1737·50 be raised so as to become Rs. 2432·50?

3. Using tables evaluate to 2 dec. places:

$$\sqrt{\frac{76.35}{(2.4)^3}}.$$

4. Find the simple interest on Rs 5050·74 for 4 years at $2\frac{1}{2}\%$.

5. Express 5 tons 17 cwt. 3 qrs. as a decimal of a ton.

6. A boy's marks in an examination are as follows:

English	55/150
History	48/60
Science	72/90.

Find (*a*) his percentage in each paper (*b*) his average percentage for the examination.

7. Find the C. I. on Rs 27500 for 2 years at 4% per annum.

8. The distance between two places on a survey map scale 1 cm to 20 km is 15·2 cm. Find how long a car travelling at 80 km per hour would take to travel from one town to the other.

9. ABC is a triangle in which $a=6$, $b=7$, $c=10$. Find the angle C.

10. A roll of wire is 10 metres long and its cross-section is circular. If the diameter of the cross-section is 0·121 cm. find the weight of the wire if the density of the metal is 8·9 gm. per cc. ($\pi=3{\cdot}142$.)

Test 3

1. (*a*) Calculate as shortly as possible:

$$(865\times479)+(591\times479)+(544\times479)$$

(*b*) $2865^2-(2565\times2865)$

(*c*) $(\frac{22}{7}\times5{\cdot}73^2)-(\frac{22}{7}\times1{\cdot}27^2)$

2. Given that $x=\frac{1}{3}a-1$ and $y=\frac{1}{4}a+1$;
prove that $3x-4y+7=0$.
Find also the numerical values of x, y and a when $x=\frac{1}{5}y$.

3. Use tables to calculate correct to 3 sig. figures

(*i*) $23{\cdot}76\times0{\cdot}0004891$.

(*ii*) $\dfrac{0{\cdot}005764\times\sqrt[3]{387{\cdot}5}}{0.0986}$

4. A man starts from town X and walks 4 km due east to a town Y. He then walks 5 km in the direction north-east to a town P. Using tables calculate (*i*) how far P is north of the line XY giving your answer in km correct to 3 sig. figures (*ii*) the size of the angle PXY to the nearest minute.

5. A man sells two houses for Rs 15960 each, gaining 5% on one cost price and losing 5% on the other; find the cost price of each and his total gain or loss per cent on the total cost price.

6. A cylindrical jug whose internal radius is 4 cm stands on a table. A solid sphere of radius 3 cm is put inside the jug. Water is then poured into the jug until the sphere is half submerged. The sphere is now taken out. How much does the water level drop? (Vol. of sphere is $\frac{4}{3}\pi r^3$.)

7. A thin hemispherical bowl is suspended from a point by three strings each 24 cm long, attached to three points on the rim of the bowl at the vertices of an equilateral triangle. The angle between each pair of strings is 90°. Calculate the diameter of the bowl.

8. Three partners go into business with capitals of Rs 3500, Rs 2300, and Rs 4700. At the end of a year they find they have made a profit of 18% of their capital. They put aside Rs 840/- for the purpose of extending their business and divide the rest between them in proportion to their respective contributions. What does each partner receive?

9. A man borrows Rs 14800 at Compound Interest and clears his debt at the end of two years. If the rate of interest charged is $2\frac{1}{2}\%$ per annum find to the nearest rupee the amount he will have to pay at the end of this period.

10. PTOQ is a rectangular face of a wedge. PQOT inclined at an angle of 44° to the plane PQRS. PQ=48 cm. The line PM

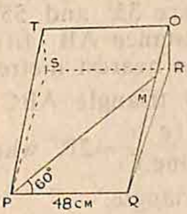


Fig. 50

in the plane PQOT makes an angle of 60° with PQ.

Calculate (*i*) the length of MQ.

(*ii*) the angle PM makes with the plane PQRS.

Test 4

1 (a) Find the value of $\dfrac{\frac{1}{27}-\frac{1}{81}}{\frac{1}{27}+\frac{1}{81}}$ of $\dfrac{\frac{2}{3}\text{ of }\frac{4}{9}}{\frac{2}{3}\div\frac{4}{9}}$.

(*b*) Find the cost of 217 articles at £3 19*s*. 11*d*.

2. A man invests £640 for 2 years at $3\frac{3}{4}\%$ simple interest. If he invests the same amount for the same time at $3\frac{1}{2}\%$ compound interest, how much more would he earn as interest?

3. A can do a piece of work in 20 days and B can do the same work in 30 days. If they are helped by a third man C they can together complete the work in $7\frac{1}{5}$ days. How long will C alone take to do the work?

4. Twenty men were employed to do a piece of work in 15 days. Some men left work and the remainder completed the work in 25 days. Find the number of men who left the work.

5. By selling a car for £275 a dealer lost 12%. For what should he have sold it to gain 12%?

6. Find the cost of papering the walls of a room 21 feet long, $16\frac{1}{2}$ feet wide, $11\frac{1}{2}$ feet high at Re 0·66 per sq. yard. (No allowance for doors etc.)

7. A thin rod of length 16 cm can just be made to fit inside a box with the lid closed. The base of the box is a square of side 10 cm internal measurements. Find the angle which the rod makes with the bottom of the box, and the internal height of the box. Give the height to the first decimal place.

8. A is a point on one side of a straight road. B and C are two points 105 metres apart on the opposite side of the road. The angles ABC and ACB are 35° and 55°, the angle BAC being 90°. Calculate (*i*) the distance AB (*ii*) the width of the road. Give your answers to the nearest metre.

9. Prove that in any triangle ABC

$$\frac{a}{\text{sine A}}=\frac{b}{\text{sine B}}=\frac{c}{\text{sine C}}=2R$$ where R is the radius of the circum-circle of the triangle.

Prove that the area of a Δ ABC is $\frac{1}{2}bc \sin A$. You may assume that the area of a Δ is $\frac{1}{2}$ base × height.

10. Two men, one cycling at 12 km per hour and the other walking at 4 km per hour, start together from a place X towards a place Y, 12 km away. At the same instant a second cyclist travelling at 9 km per hour starts from Y to meet them. By means of a graph find how many minutes elapse between the times of meeting of the second cyclist with the other 2 men. Take 1 cm to represent 1 km and 2 cm to represent 20 minutes.

Test 5

1. Find the price of 6537 articles at Rs 99·90 each.

2. A candidate appears for an examination and obtains the following marks:

English	63%
Physics	75%
Chemistry	84%.

What percentage does he get on the whole?

3. Find the radius of a circle of area 32 sq. cm, giving your answer correct to 2 sig. figures. $\pi = \frac{22}{7}$.

4. A man bought envelopes at the rate of Rs 40·50 a thousand

and sold them at the rate of Rs 2 a packet of 25. Find his percentage profit correct to 4 sig. figures.

5. Of two taps one can fill a cistern in two hours and the other in three hours. They are opened together, and after half an hour it is found that they have filled 35 gallons. Find the capacity of the cistern.

6. What principal will amount to £4047 in $3\frac{1}{2}$ years at 4% simple interest ?

7. How many cubic cm of wood will be required to make a lidless box of external dimension 8 cm by 7 cm by $5\frac{3}{4}$ cm and thickness 0·25 cm. Find the weight of the box if the density of wood is 0·6 gm. per c.c.

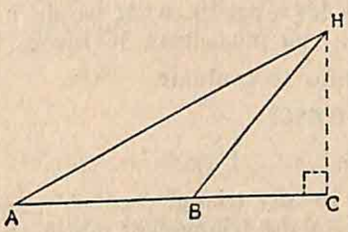


Fig. 51

8. In the above given figure angle BCH=90°, HB bisects angle AHC, AC=30 feet, and CH=10 feet. Using your tables, calculate angle AHC to the nearest minute and the length of HB correct to one decimal place.

9. Use tables to evaluate the following:

(*i*) $6{\cdot}248 \times \sqrt{\frac{17{\cdot}6}{32.2}}$ (*ii*) $\sqrt[3]{0.76}$.

10.

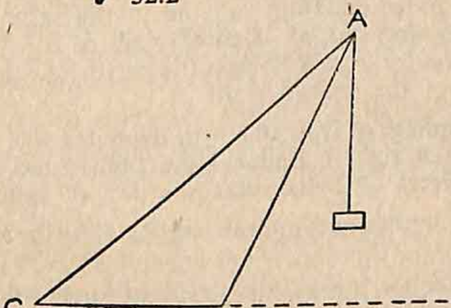


Fig. 52

The given figure represents a crane AC=21 feet, the jib AB=15 feet, BC=9 feet. Prove that angle ABC is 120°, using the cosine formula. Calculate (*i*) the height of A above the level ground CB (*ii*) the angle ACB.

Test 6

1. A rug 72 inches long and 36 inches wide is made of wool. The weight of wool used is $10\frac{1}{4}$ lb. What weight of wool is used per sq. foot of the rug? Give your answer in ounces to 2 dec. places.

If the wool costs $18\frac{2}{3}$*s*. per lb. what would be the cost of wool used for making a rug measuring 36 inches by 18 inches?

2. Use logarithms to evaluate

(*i*) $0{\cdot}0123 \div 0{\cdot}4567$

(*ii*) $\sqrt{0{\cdot}0246}$.

3. Prove that, in any triangle, $a^2=b^2+c^2-2bc \cos A$. Find the smallest angle of the triangle of which the sides are 15, 17, and 24 cm.

4. The perimeter of a quadrant is 200 m. Find the area in sq. metres.

5. Tea at Rs 3 per lb. is mixed with tea Rs $5\frac{1}{3}$ per lb. so that a profit of 50% is made by selling the mixture at Rs $6\frac{1}{2}$ per lb. In what ratio are the teas mixed?

6. A train 80 metres long moving at the rate of 80 km per hour overtook a second train moving in the same direction at 56 km per hour and passed it completely in $22\frac{1}{2}$ sec. Find (*i*) how long the first train took to pass the driver of the second train (*ii*) the length of the second train.

7. A solid sphere of iron 10 cm in diameter was melted down and recast into a right cylinder 3 cm high. Find the diameter of the base correct to 2 dec. places. Vol. of sphere$=\frac{4}{3}\pi r^3$.

8. Find the compound interest on Rs 600 for 3 years at 4% per annum.

9. A retail seller marks his goods at such prices that 35% of the marked price is his profit. At what price will he mark an article which cost him Rs 78? If he is prepared to sell at a price

which yields him 30 per cent profit on his outlay at what price will he be prepared to sell an article marked Rs 100?

10. If A is an angle between 0° and 180° such that sine A $=\frac{11}{61}$, find the possible values for cos A and tan A.

Test 7

1. (*a*) Simplify $7\frac{1}{2}\times\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$.

(*b*) Multiply £4 2*s*. $7\frac{1}{2}$*d*. by 17.

(*c*) Express 90 m.p.h. in yards per minute.

2. Simplify without using tables

(*i*) $36 \times (2^8 \times 3^2)^{\frac{1}{2}}$

(*ii*) $2^{-5} \times 3^{-8} \times 8^{\frac{2}{3}}$.

3. If $a=\sqrt{2rh-h^2}$ find a, using log tables, if $r=4{\cdot}853$ and $h=2{\cdot}762$.

4. If a train 200 metres long overtook a cyclist travelling at 8 km per hour on a parallel road and passed him in 15 sec., find the speed of the train in km per hour.

5. Find the compound interest on £935 at $1\frac{1}{2}$% for 4 years.

6. A, B and C go into partnership and buy a business. A provides $\frac{4}{15}$ of the capital, B provides Rs 11400, and C Rs 15000. If they share profits in proportion to their investments, how much will each get in a year when Rs 3,000 profit is made?

7. A tap running into a cistern can fill it in $1\frac{3}{4}$ hours and a tap from the cistern can empty it in $5\frac{1}{3}$ hours. If both taps run for $2\frac{1}{4}$ hours the cistern gains 10800 litres. Find how many litres per minute run through each tap.

8. The price of a certain article was increased by Rs 3. In consequence, two fewer articles could be bought for Rs 180 than had previously been possible. Find the original price of the article.

9. A vertical pole is erected on a piece of level ground. A man whose eye is 5 feet above the ground is standing 40 feet from the foot of the pole. He observes that the pole subtends an angle of 50° at his eye. Calculate with the use of tables the height of the pole above the ground correct to the nearest integer.

10. A road sign is a vertical equilateral triangle with its base parallel to the ground. When the sun is directly behind the triangle and at an altitude of 75° the shadow cast is an isosceles triangle. Calculate the angles of this triangle

Test 8

1. Calculate as shortly as possible

(*a*) $(726 \times 245) + (871 \times 245) + (403 \times 245)$

(*b*) $3896^2 - (3596 \times 3896)$

(*c*) $(\frac{22}{17} \times 14\cdot52^2) - (\frac{22}{17} \times 2\cdot48^2)$.

2. Use tables to calculate the following correct to 3 sig. figures:

(*a*) $\sqrt{0.05891}$ (*ii*) $2 \times 3\cdot142 \sqrt{\frac{13\cdot6}{981}}$.

3. In an examination, 1200 candidates were Indian and 700 were foreigners. 40% of the Indians and 30% of the foreigners passed. What percentage of the whole number of candidates failed?

4. A cyclist starts at 11 a.m., riding at 10 km an hour, for a place 12 km away, until he has a puncture. He then spends 20 min. trying to mend it; but being unable to do so he walks the remaining distance at 3 km per hour, arriving at his destination at 1.30 p.m. Find by a graphical method how far he has cycled before he has the puncture.

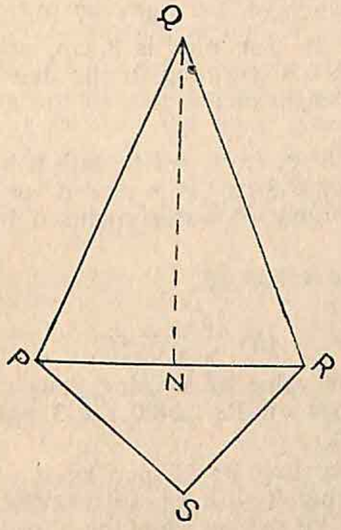
5. A pyramid on a square base of side 10 inches is 1 foot in height. Calculate.

(*i*) its total surface area,

(*ii*) the length of a slant edge,

(*iii*) the inclination of a face to the base.

6. A man rolls a ground of 7 acres with a mechanical roller 7 feet wide and 28 inches in diameter. If the ground is covered, over how many miles does the roller travel and how many revolutions does it make? You may ignore the turns. $\pi = \frac{22}{7}$.

7. 

PQRS is a quadrilateral.
$\angle QPR = \angle QRP = 67°$.
$\angle$ PR = 10 cm.
$\angle PRS = 48°$.
$\angle PSR = 90°$; QN is perpendicular to PR.

Fig. 53

Calculate (*i*) PS (*ii*) PQ (*iii*) QN.

8. A rectangular block of iron is 5 inches long, 5 inches wide and $1\frac{1}{2}$ inches thick and weighs 84 ounces. It is drawn out into a wire of diameter 0·16 in. Find the weight of 45 feet of the wire. $\pi = 3{\cdot}142$. Give your answer correct to 3 significant figures.

9. A regular polygon of ten sides is inscribed in a circle of 2 cm radius. Calculate, using tables, (*i*) the length of one side of the polygon (*ii*) the area of the polygon.

10. ABC is a triangle in which AB = 6 cm, BC = 7 cm, and angle ABC is 60°. Find by calculation the area of the triangle.

Test 9

1. A boy spends 3/10 of his money in the first week, and 3/7 of the remainder the following week; he has then Rs 23·48 left. What did he start with?

2. Find the percentage increase in the area of a rectangle if each side is increased by 15%. Find the percentage decrease in area if each side is decreased by 10%.

3. If $W=14x^2\sqrt{y}$, find W when $x=5/6$ and $y=7/10$.

4. The internal diameter of an iron pipe is 8 cm, and the metal is 5 mm thick; the pipe is 120 cm long. If the density of iron is 7·2 gm per cc, find the weight of the pipe to the nearest ½ kg.

5. A sheet of metal of area 8 sq. cm is rolled into a hollow cylinder and its internal diameter is 3 cm. It is placed vertically in a tub of water. Find the volume of water enclosed by the cylinder.

6. (*a*) Using tables, find the value of

(*i*) $10^{1\cdot5869}$ (*ii*) $\sqrt[3]{0\cdot0621}$

(*b*) If $10^x=15\cdot21$, find the value of x using your tables.

7. Find the compound interest on Rs 12400 for 3 years at 5% correct to one decimal place.

8. Full-cream milk-powder costing Rs 18 per kg is mixed with skimmed-milk powder costing Rs 4 a kg so that the mixture may be sold at Rs 20 per kg thereby making a profit of 25%. Find the ratio in which the milk powders are mixed.

9.

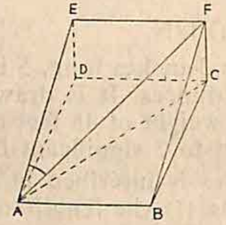


Fig. 54

ABCDEF is a wedge.
Angle FBC=18°.
Angle EAF=35°.
Find the angle FAC.

10. Find the area of a regular five-sided polygon inscribed in a circle of radius 10 inches.

Test 10

1. A tourist with £119 stays in a London hotel and pays £3 7*s*. 6*d*. per day for board and lodging. How many days can he stay and how much will he have left over?

2. Three metal spheres of radius 3 cm, 2 cm and 1 cm are melted and recast into a single sphere. Find the radius of this sphere.

3. Find the sum which amounts to Rs 400·50 in $2\frac{1}{2}$ years at $4\frac{1}{2}\%$ per annum at simple interest.

4. A cake is cut into two pieces in the ratio of 2 : 3. The smaller piece is again cut into two pieces in the ratio of 4 : 5. Find the ratio of the three pieces to each other.

5. The maximum marks for three papers in an examination are 200, 300, 250 and a candidate obtains 50%, 60% and 58% on them respectively. What is his percentage score on all three papers together?

6. A courtyard 10·08 metres long and 3·36 metres wide is paved with stones 9 cm by 7 cm. How many stones are required? If the stones are sold at Rs 25 per thousand, find the cost of paving the courtyard.

7. A box with a lid is made of wood 1 cm thick. The external dimensions are: length 3 dm, breadth 2·1 dm and height 1·6 dm. Find the volume of the wood used in cubic centimetre.

8. Draw a graph which can be used to convert centigrade temperatures to Fahrenheit. Use this graph to convert 76°C to F and 104°F to C.

9. Use tables to evaluate

(*i*) $(1{\cdot}32)^3+(7{\cdot}38)^{\frac{1}{3}}$

(*ii*) $\sqrt{\dfrac{27{\cdot}49}{4{\cdot}35\times 9{\cdot}73}}$.

10. ABCD is a parallelogram in which AB = 6 cm, BC = 4 cm and angle A = 117°. Find the area of the parallelogram.

ANSWERS

EXERCISES 1. PAGE 1.

1. (a) 36 216 | (b) 48 288 | (c) 7 1260 | (d) 17 78540 | (e) 88 15300

2. (a) 675
(b) 8316
(c) 2184
(d) 4760
(e) 2772
(f) 3420
(g) 2415
(h) 1120
(i) 30576

3. £117 15s. 4. 108 5. 315 6. 112
7. 318 8. 80 9. 32 10. 10.09 a.m.
11. 685 12. 49 13. 1 min; 12,10,5,4.
14. 7,140; 35,28.

EXERCISES 2. PAGE 3.

1. 1
2. $\frac{57}{65}$
3. $\frac{3}{32}$
4. $2\frac{1}{16}$
5. $7\frac{29}{30}$
6. $\frac{47}{77}$
7. $\frac{37}{90}$
8. $\frac{1}{45}$
9. $3\frac{8}{9}$
10. $4\frac{2}{3}$
11. $3\frac{3}{4}$
12. $1\frac{1}{2}$
13. $\frac{3}{10}$
14. $\frac{11}{7}$
15. $\frac{5}{16}$
16. $\frac{3}{8}$
17. $30\frac{3}{44}$
18. $\frac{29}{210}$
19. 3
20. $\frac{35}{446}$

EXERCISES 3. PAGE 4.

1. (*a*) 2s.
(*b*) £1 6*s*. 3*d*.
(*c*) Rs 137·25
(*d*) 10hrs. 10 min. 2 sec.
(*e*) 145 m 60 cm.

2. (*a*) $\frac{3}{4}$
(*b*) $\frac{17}{40}$
(*c*) $\frac{3}{4}$
(*d*) $\frac{3}{5}$
(*e*) $\frac{5}{12}$
(*f*) $\frac{17}{72}$
(*g*) $\frac{1}{480}$
(*h*) $\frac{3}{40}$

3. Rs 7·50 4. 120 5. $\frac{17}{6}$ 7. $7\frac{1}{2}$
8. £5 9. 91 litres 10. £450 11. Rs. 180
12. Rs 2800 13. 60 acres 14. 900 litres 15. Rs. 2240

EXERCISES 4. PAGE 6.

1. $\frac{17}{50}$; $\frac{23}{40}$; $\frac{3}{8}$; $\frac{7}{8}$; $\frac{5}{16}$; $\frac{223}{2000}$; $\frac{49}{80}$.
2. 0·750; 0·929; 0·667; 0·444; 3·142; 3·143;
0·571; 0·308; 0·364; 0·733; 0·840; 0·819.

EXERCISES 4 (contd.)

3. $57\frac{1}{7}\%$; $66\frac{2}{3}\%$; 60%; 47%; $84\frac{8}{13}\%$; 76%;
$45\frac{5}{11}\%$; $62\frac{6}{37}\%$; $41\frac{3}{13}\%$; 53%; $17\frac{7}{10}\%$; $69\frac{168}{173}\%$.
4. $\frac{2}{3}$; $\frac{11}{20}$; $\frac{1}{8}$; $\frac{6}{11}$
$\frac{3}{700}$; $\frac{1}{275}$; $\frac{4}{7}$; $\frac{8}{13}$
$\frac{11}{30}$; $\frac{14}{25}$; $\frac{79}{135}$; $\frac{5}{17}$
$\frac{1}{12}$; $\frac{4}{9}$; $\frac{12}{23}$; $\frac{1}{1000}$
$\frac{18}{25}$; $\frac{9}{125}$.
5. (*a*) $\frac{3}{8}$; $\frac{7}{19}$; $\frac{4}{11}$; $\frac{1}{3}$; $\frac{2}{7}$ (*b*) 0·4; $\frac{3}{8}$; 36%
(*c*) $\frac{1}{3}$; 33; $\frac{13}{40}$; $\frac{8}{25}$ (*d*) $\frac{22}{7}$; 3·1416; $\frac{355}{113}$
(*e*) $\frac{25}{31}$; $\frac{9}{20}$; $\frac{17}{39}$; $\frac{8}{19}$ (*f*) 0·756; 75%; $\frac{187}{250}$.
(*g*) $\frac{1}{3{\cdot}142}$; 0.318 (*h*) $\frac{4}{3{\cdot}7}$; $\frac{2}{25}$
(*i*) $7\frac{1}{2}$; $7\frac{2}{5}$; 7·39; $7\frac{5}{13}$; $7\frac{4}{11}$ (*j*) $\frac{1}{3}$; 0·33; $\frac{12}{37}$; $\frac{11}{34}$; 32%
(*k*) 0·87; $\frac{38}{45}$; $\frac{59}{70}$; $\frac{16}{19}$; $\frac{21}{25}$.
6. (*a*) 0·6 (*b*) 0.75 | (*c*) 0·0375 (*d*) 0·55 | (*e*) 0·26
7. 0·436 | 10. 0·125; 0·28 | 13. 0·15; 0·05
8. 0·955 | 11. 0·288 in. | 14. 227 litres
9. 0·261 | 12. 0·02 | 15. 12·50
16. 2414 m. 17. 11/40 18. 5/11
19. (*a*) 128 tons 9 cwt. 2qr. (*b*) £231 4*s*. 3*d*.
(*c*) Rs 137·25. (*d*) 10 hr. 10 min. 25 sec.
(*e*) 113 yd^2 7 ft.^{2}
20. Rs 8.32 21. 35 tons 11 cwt. 2 qr. 22. Former; £7 11*s*
23. $\frac{108}{221}$ Rs 36·40, Rs 35·70; Rs 72·10.
24. (*b*) £53 16*s*. 3*d*.
(*c*) £32 6*s*. 4*d*.
(*d*) £49
(*e*) £72 1*s*. 3*d*.
(*f*) £64 15*s*. 1*d*.
(*g*) 5 tons 14 cwt. 3 qrs.
(*h*) 7 tons 7 cwt.
(*i*) 7 hrs. 6 min. 43 sec.
(*j*) 73 hrs. 15 min. 22 sec.

EXERCISES 4 (contd.)

25. (*a*) $\frac{23}{30}$
(*b*) $\frac{3}{4}$
(*c*) $\frac{15}{19}$
(*d*) $\frac{25}{52}$

EXERCISES 5 PAGE 12.

1. (*a*) 44 tons 13 cwt.
(*b*) 110 tons 10 cwt.
(*c*) 12 cwt. 73 lb.
(*d*) 231 tons 13 cwt. 3 qrs.
(*e*) 388 tons 2 cwt. 2 qrs.
2. £553 3*s*. 9*d*.
3. £139 15*s*. ½*d*.
4. £85 8*s*. 2*d*.
5. 21·3 miles
6. 131·4 km.
7. (*a*) 246 hrs 7min. 55 sec.
(*b*) 227 hrs. 15 min.
(*c*) 413 hrs. 47 min.
(*d*) 253 hrs. 20 min.
(*e*) 374 yds. 11 in.
(*f*) 430 yds.
(*g*) 1166 yds. 9 in.
8. £925 6*s*. 3*d*.
9. (*a*) £1 19*s*. 10½*d*.
(*b*) 342 lb. 9 oz.
(*c*) £8 18*s*. 9*d*.
(*d*) £64 17*s*. 6*d*.
10. Rs 2·19.
11. 231 tons. 12 cwt. 3 qrs.
12. £3 4*s*. 4*d*.; £4 3*s*. 2*d*.; £3 7*s*. 5*d*.; £3 18*s*.; £2 2*s*. 11*d*. £4 16*s*. 8*d*.

EXERCISES 6 PAGE 13.

1. £11
2. £38 13*s*. 6*d*.
3. £14 2*s*. 9*d*.
4. Rs 1471·10
5. Rs 1530·25
6. (*a*) Rs 5663·13
(*b*) Rs 3459·38
(*c*) Rs 7943·75
(*d*) Rs 722,843·13
(*e*) Rs 437,034·38
7. £9 10*s*. 11*d*.
8. (*a*) £12 17*s*. 3*d*.
(*b*) £30 14*s*. 2*d*.
(*c*) £121 6*s*. 11*d*.
(*d*) £376 3*s*. 7*d*.
(*e*) £517 6*s*. 10*d*.

EXERCISES 6 (contd.)

9. 970 lb. $9\frac{5}{12}$ oz.
10 $2\frac{17}{64}$ oz.
11. Rs 62·91
12. Rs 58·44

EXERCISES 7. PAGE 17.

1. 23
2. 27
3. 29
4. 32
5. 39
6. 47
7. 53
8. 71
9. 78
10. 85
11. 99
12. 253
13. 313
14. 2070
15. 5001
16. 91·35
17. 26·05
18. 86·7
19. 29·94
20. 2·64
21. 5·4
22. 2·3
23. 2·7
24. 3·2
25. 0·28
26. 0·152
27. 0·0438

EXERCISES 9. PAGE 22.

1. (*a*) 8·92 in. (*b*) 2·82 cm (*c*) 0·57 km
(*d*) 0·25 ft (*e*) 0·79 m (*f*) 2·5 cm
(*g*) 92·40 cm (*h*) 892 m (*i*) 89·3 in.
2. (*a*) 13 cm (*b*) 9 cm (*c*) 7·28 cm
(*d*) 7·75 cm (*e*) 12·55 cm
3. 8·66 cm
4. 8·78 ft
5. 25·5 ft
6. 311 yds; Rs 7480
7. 4·5 ft, 22·3 ft
8. 21·4 ft
9. 12·5 cm
10. 24·5 ft
11. 20·8 cm
12. 27·7 cm
14. 48·5
15. 3·83 cm
16. 3·99 cm
17. 6·75
18. 9·43 cm
19. 12·4 in.
20. 7·21 km
21. (*a*) $1\frac{2}{5}$ (*b*) 1·47 (*c*) 1·86 (*d*) 1·9 (*e*) $1\frac{1}{8}$ (*f*) $3\frac{2}{3}$
(*g*) 3·38 (*h*) 0·36 (*i*) $1\frac{1}{2}$ (*j*) $1\frac{1}{4}$ (*k*) $1\frac{2}{5}$ (*l*) $\frac{9}{35}$
22. (*a*) 36·79 (*b*) 9·49 (*c*) 5·67 (*d*) 2·24 (*e*) 6·71 (*f*) 37·36

EXERCISES 10. PAGE 26.

1. (*a*) $4\frac{113}{720}$ sq. ft (*b*) 18 sq. ft 74 sq. in. (*c*) 10 (*d*) 77
(*e*) $195\frac{8}{9}$ (*f*) $58\frac{29}{32}$ (*g*) 22·22 (*h*) 0·1605
(*i*) 45·485 (*j*) 146·200 (*k*) 1197×10^{8} (*l*) 1,451,160

Exercises 10 (contd.)

2. (*a*) 85 ft (*b*) 3 ft. 4 in. (*c*) 250 yds. (*d*) 253 yds.
(*e*) 3 ft. $6\frac{1}{2}$ in. (*f*) 11·1 m (*g*) 8·9 m (*h*) 79·6 m.
3. (*a*) 768 sq. ft. (*b*) 852 sq. ft.
(*c*) $1006\frac{1}{4}$ sq. ft. (*d*) $1069\frac{1}{4}$ sq. ft.
4. Rs 33·33
5. (*a*) Rs 396·80 (*b*) Rs 513·$82\frac{1}{2}$ (*c*) Rs 528·1
6. 14*s*. $9\frac{1}{2}$*d*.
7. (*a*) 11 ft (*b*) $30\frac{1}{2}$ ft (*c*) $28\frac{3}{4}$ ft (*d*) 27; $13\frac{1}{2}$ ft
(*e*) $25\frac{1}{4}$; $20\frac{1}{4}$ ft
8. 14 ft 9. $12\frac{1}{2}$ ft 10. Rs 218·75
11. Rs 11·81 12. Rs 2·25 13. 66 fr.
14. £38 10*s*. 15. Rs 525 16. 0·9 acres

Exercises 11. Page 28.

1. (*a*) $55\frac{1}{8}$ (*b*) $39\frac{27}{32}$ (*c*) $9\frac{1}{2}$ (*d*) 396·9 (*e*) 3·6
(*f*) 4 (*g*) ·57 (*h*) 4 (*i*) $9\frac{1}{2}$ (*j*) 29·8
(*k*) 9 (*l*) 11
2. 20·7 ft.^{2} 3. Rs 10·24 4. Rs 2·41
5. 13 6. 1*s*. $3\frac{1}{4}$*d*. 7. Rs 31·25
8. Rs 72 9. 6 10. 375
11. 96 12. 82·35 lb.
13. (*a*) 3·7 (*b*) 17·6 (*c*) 10·3 (*d*) 8·2
14. 0·65 15. 32 lb. 12 oz.
16. (*a*) 208·6 (*b*) 134·01 (*c*) 5·6 (*d*) 3·8
(*e*) 11·06 (*f*) 717·75 (*g*) 1022·4
17. 0·23 18. 0·1 in. 19. 1 ft. $1\frac{1}{2}$ in. 20. 0·087

Exercises 12. Page 31.

1. 1·17 ft. 2. 449·3 3. 2140·3 4. 2·7
5. 2·19 ft. 6. 5·44 ft. 7. 4 in. 8. 2·56 in.
9. $187\frac{1}{2}$ min. 10. In $62\frac{1}{2}$ min. 11. $53\frac{1}{3}$ min.

EXERCISES 13. PAGE 35.

1. 144 2. 144 3. $26\frac{1}{4}$ in. 4. £55
5. Rs 346·50 6. 154 sq. in. 7. 369·985 sq. ft.
8. £40 6*s*. 8*d*. 9. 3229 fr. 6 cent. 10. 140·3 sq. cm.
11. The square by 0·302 sq. in.
12. Square by 8·1225 sq. cm
13. Circle by 8·25 sq. in.
14. Square by 2·9 sq. cm
15. 31·8 sq. in. the circle. By 6·8 sq. cm
16. 79·07 sq. cm 17. 50·3 sq. inch 18. 63·7 m. 112·9 m.
19. 3 cm 20. 7·43 sq. cm 21. Rs. 4564·75
22. 360 gm 23. 5 cm 24. 16·75 sq. cm
25. The radius = 6 cm (*a*) 28·26 sq. cm (*b*) 47·12 sq. cm
(*c*) 33·92 sq. cm (*d*) 75·39 sq. cm

EXERCISES 14. PAGE 39.

1. 7 in., 220 sq. in., 38·5 sq. in.
2. 22 in., 12 in., 264 sq. in.
3. (*a*) 176 sq. in. (*b*) 211.2 sq. in. (*c*) 105·6 sq. cm
(*d*) 314 sq. cm (*e*) 113.04 sq. in.
4. (*a*) 88 sq. cm (*b*) 471 sq. cm (*c*) 132 sq. ft
5. 3·5 cm
6. (*a*) 47·1 sq. ft (*b*) 235 sq. ft (*c*) 47·1*x* sq. ft
7. 286 sq ft
8. 251·4 cm
9. (*a*) 385 cu ft (*b*) 231 c.c. (*c*) 88·55 cu. c.
(*d*) 15·4 cu dm (*e*) 154 c.c.
10. (*a*) 5·25 cm (*b*) 5·44 cm (*c*) 11·76 in.
(*d*) 14·5 ft
11. 4 ft. 12. 8·87 kg 13. 7 in. 14. 273 lb.
15. 1·82 ft 16. 29·3 in. 17. 0·087 in. 18. 183 min.
19. 8·4 in. 20. 1·42 cm

EXERCISES 15. PAGE 42.

1. (*a*) 0·96 (*b*) 15·62 (*c*) 2·77 (*d*) 3·62
2. 0·21; 3·42; 0·606; 0·79 3. 20·8 cm.
4. 18 cm. 5. 132·6 cm.

EXERCISES 16. PAGE 43.

1. 48 c. ft. 2. 10·3 in. 3. 232·35 in.^{3}
4. 96 sq. ft. 5. 150·5 sq. in.
6. (*i*) 174·75 cu. in. (*ii*) 407·3 cu. in.
(*iii*) 869·6 cu. in. (*iv*) 640 cu. in.

EXERCISES 17. PAGE 43.

1. 770 sq. ft; Rs 34·22 2. 1,480 sq. ft
3. 11·25 sq. in. 4. 36·3 sq. in.
5. 3,181$\frac{9}{11}$ sq. yds.; 2,432$\frac{8}{77}$ sq. yds.; Rs 608·03
6. 50,575 sq. ft; 5,225 sq. ft 7. 525·6 sq. in.
8. 60·41 sq ft. 9. 41·286 sq. in. 10. 1089 : 430911
11. 1896 : 5329 12. 2209 : 1213 13. 3:11 15. $\frac{9}{14}$
16. 18·7 gm 17. 0·5 lb. 18. 0·3 lb.
19. 0·75 lb. 20. 10·7 gm

EXERCISES 18. PAGE 48.

1. (*b*) $\frac{5}{8}$ (*c*) $\frac{6}{8}$ (*d*) $\frac{7}{10}$ (*e*) $\frac{88}{81}$
(*f*) $\frac{11}{8}$ (*g*) $\frac{81}{105}$ (*h*) $\frac{y}{x}$ (*i*) $\frac{x}{y}$.
2. (*a*) $\frac{7}{5}$ (*b*) $\frac{9}{4}$ (*c*) $\frac{13}{11}$ (*d*) $\frac{5}{7}$
(*e*) $\frac{7}{12}$ (*f*) $\frac{25}{23}$ (*g*) $\frac{8}{5}$ (*h*) $\frac{11}{4}$
(*i*) $\frac{2}{5}$ (*j*) $\frac{9}{4}$ (*k*) $\frac{27}{8}$

EXERCISES 19. PAGE 54.

1. 10 days 2. 21·56 sec. 3. 100 baskets
4. 46480 lb. 5. 4 gal. 6. 1042 lb.
7. Rs 64·26 8. Rs 236·50 9. £23 19*s*. 6*d*.
10. 17 11. 35 12. £31 12*s*. 3*d*.
13. (*a*) 43 (*b*) 91 14. 205

EXERCISES 19 (contd.)

15. 54
16. Oct. 3rd at 6 p.m.
17. 35
18. 8 days
19. £67 15*s*. 8*d*.
20. Rs 59·40
21. Rs 1782·50
22. 1632
23. Rs 19·80
24. Rs 199
25. Rs 76
26. Rs 60
27. 10·88 lb.
28. 33 min. (to nearest min.)
29. 690 fr. 90 cent.
30. 105·4 gm (nearest $\frac{1}{10}$ gm)
31. 17384 kg
32. 42
33. 11·2 ft
34. 8·55. cm; 29·94 sq. cm
35. (*a*) 21·3 cm.; 351·7 sq. cm
    (*b*) 10 cm. 40·2 sq. cm
    (*c*) 23·5 cm.; 1473 sq. cm
    (*d*) 13·5 cm.; 38·3 sq. cm
36. (*a*) 67·6° (*b*) 148·9°
    (*c*) 129·1° (*d*) 127·1°.

EXERCISES 20. PAGE 58.

1. 6 hrs.
2. 18
3. 45
4. 5 hrs.
5. 15
6. 40
7. 40 days
9. 45
10. 27 days
11. 5 days
12. 16 days
13. $7\frac{1}{2}$ days
14. $9\frac{1}{2}$ days
15. 9 days
16. $1\frac{1}{2}$ days
17. 8 days

EXERCISES 21. PAGE 61.

1. Rs 303·75
2. 15 days
3. Rs 10
4. Rs 350
5. 60
6. 3·27 kg
7. 243
8. 12
9. Rs 3,751
10. Rs 378
11. Rs 180
12. 18
13. 9
14. 6
15. 27

EXERCISES 22. PAGE 63.

1. 24: 32: 48
2. 408: 612: 714
3. 159; 159; 265
4. 16; 18; 22; 26
5. 20; 24; 28
6. $43\frac{1}{5}$; $28\frac{4}{5}$
7. 48; 32; 24
8. 96; 320; 120
9. 954; 1219
10. 735; 822; 1330
11. $\frac{2}{7}$
12. $\frac{5}{156}$
13. $183\frac{3}{4}$; $229\frac{11}{16}$; $321\frac{9}{16}$
14. $309\frac{78}{83}$; $247\frac{79}{83}$; $177\frac{9}{83}$
15. 42; 54
16. $68\frac{4}{7}$ degrees

EXERCISES 22 (contd.)

17. $140\frac{20}{77}°$; $93\frac{39}{77}°$; $70\frac{10}{77}°$; $56\frac{8}{77}°$
18. 1·2; 2·3; 0·5
19. 44·4; 27·6; 15·6
20. 164·5 cm; 366·6 cm
21. 255·3
22. $4·1\frac{1}{3}$ cm; $3·3\frac{1}{3}$ cm; $3·7\frac{1}{3}$ cm
23. Rs 766; Rs 383; Rs 1532
24. Rs 36·45; Rs 12·15; Rs 48·60
25. Rs. 52·22
26. Rs 46·52; Rs 46·52;
    Rs 58·15; Rs 58·15
27. $787\frac{1}{2}$; 350; 462 acres
28. $16\frac{1}{2}$; 22; $27\frac{1}{2}$
29. 270; 450; 825
30. £87 12*s.*
    £205 8*s.*
    £711 0*s.*
31. Rs 474·64
    Rs 1,107·50
    Rs 189·86
32. Rs 126; 162; $175\frac{1}{2}$

EXERCISES 23. PAGE 65.

2. (*a*) 1 : 1584
   (*b*) 1 : 1056
   (*c*) 1 : 10560
   (*d*) 1 : 20000
   (*e*) 1 : 6336
   (*f*) 1 : 26400
   (*g*) 1 : 500000
   (*h*) 1 : 2320
   (*i*) 1 : 736·15
   (*j*) 1 : 633600
   (*k*) 1 : 42240
   (*l*) 1 : 112640
4. 220 ft
   880 yds.
   10 ft
   $\frac{2}{3}$ ft
5. $305\frac{1}{2}$ miles
   $\frac{3}{4}$ in.
6. $3\frac{7}{16}$ miles
7. 3·4 in.
8. 400 sq. km
9. 0·99 acre
10. $1\frac{1}{2}$ sq. miles
11. $5\frac{5}{9}$ sq. miles
12. $444\frac{4}{9}$ sq. ft
13. 807 acres
14. 1·68 sq. miles
15. 36:25
16. The second
    ratio, $\frac{2}{3}$

EXERCISES 24. PAGE 67.

1. 70 gm.
3. 12·5 oz.
4. $2\frac{32}{39}$% or 2·8%; $1336\frac{4}{11}$
5. $59\frac{13}{93}$% or 59·14%
   $24\frac{68}{93}$% or 24·73%
   $16\frac{4}{31}$% or 16·13%
6. 1925, 784, 525
8. 32·10%
   38·05%
   29·85%
   810:640:251
11. 53·02%
2. $41\frac{3}{7}$% =41·43% Aniline oil
   $2\frac{8}{21}$% =2·38% P. ch.
   $21\frac{3}{7}$% =21·43% D. W.
   $31\frac{19}{21}$% =31·90% H. C.
   $2\frac{6}{7}$% = 2·86% C. C.
7. 5·02; 3·27
9. 32·40%
   34·13%
   33.47%
10. 62·8%
12. 9·6%

Exercises 25. Page 69.

1. Rs 637·50
   Rs 425
   8½%
2. Rs 112·50

3. 3,656 fr. 25 cents.
   6,896 fr. 25 cents.
   4,725 fr. 0 cent.
   11¼%
4. $ 1,608 25 *c*.
   $ 1,462 50 c.
   $ 1,755 0 c.

5. £2,407 10*s*.
   £ 722 5*s*.
   £1,083 7*s*. 6*d*

6.

Partners	Capital	Profits: 1st year	Profits: 2nd year
A	£7,200	£482 12 0	£617 10 0
B	£4,500	£301 12 6	£385 18 9
C	£5,400	£361 19 0	£463 2 6
Totals	£17,100	£1,146 3 6	£1,466 11 3

7.

Partners	Capital	Profits: 1st year	Profits: 2nd year
A	Rs. 54,000	Rs. 3,690	Rs. 5,184
B	Rs. 75,000	Rs. 5,125	Rs. 7,200
C	Rs. 63,000	Rs. 4,305	Rs. 6,048
Totals	Rs. 1,92,000	Rs. 13,120	Rs. 18,432

8.

Partners	Capital	Profits: 1st year	Profits: 2nd year
A	Rs. 63,000	Rs. 9,450	Rs. 12,600
B	Rs. 38,000	Rs. 5,700	Rs. 7,600
C	Rs. 49,000	Rs. 7,350	Rs. 9,800
Totals	Rs. 1,50,000	Rs. 22,500	Rs. 30,000

EXERCISES 26. PAGE 72.

1. (*a*) 40·6
(*b*) $14\frac{2}{7}$
(*c*) 1·36
(*d*) 1·2
(*e*) 9·75
(*f*) $57\frac{1}{7}$
(*g*) 89
(*h*) 12·8
(*i*) 229·2
(*j*) 3·7
(*k*) 4·17
(*l*) 23
(*m*) 11·8
(*n*) 86·4
(*o*) 26·26
(*p*) 1312·5
(*q*) 6000
(*r*) 0·01359
(*s*) 15·63
(*t*) 0·01203

3. 1·403
1·199
2·392
0·3397
0·324
1·431
0·428
0·181

4. (*a*) yes
(*b*) no
(*c*) yes
(*d*) no
(*e*) yes
(*f*) yes

EXERCISES 27. PAGE 74.

1. (*c*) Rs 4
(*d*) Rs 4·67
(*e*) Rs 13·33
(*f*) Rs 17·33
(*g*) Rs 20
(*h*) Rs 70·67
(*i*) Rs 0·50
(*j*) Rs 0·75
(*k*) Rs 45
(*l*) Rs 66·67
(*m*) Rs 73·33
(*n*) Rs 105
(*o*) Rs 1333·33
(*p*) Rs 13,333·33
(*q*) Rs 133; Rs 33
Rs 333·33

2. (*b*) 11*s.* $7\frac{1}{2}$*d.*
(*c*) 15*s.*
(*d*) £1 2*s.* 6*d.*
(*e*) £1 10*s.*
(*f*) £3 15*s.*
(*g*) £7 10*s.*
(*h*) £12 0*s.* 9*d.*
(*i*) £13 3*s.* $7\frac{1}{2}$*d.*
(*j*) £18 15*s.*
(*k*) £23 11*s.*
(*l*) £75
(*m*) £7500

EXERCISES 28. PAGE 80.

1. 5400 2. 2700 3. $613\frac{7}{11}$ 4. 30
5. 88 6. 3520 7. $2\frac{2}{3}$ 8. $34\frac{2}{9}$
9. $7\frac{7}{9}$ 10. 2016 11. $497\frac{7}{9}$ 12. 2
13. 0.000035 14. $1\frac{7}{18}$ 15. 750 16. 192
17. 240 18. 52·4 19. 9:8 20. 25:16
21. $2\frac{6}{11}$ (steamer) 22. Add on $\frac{3}{5}$ sec. 23. $31\frac{1}{2}$ m.p.h.
24. (*a*) 57 m.p.h. (*b*) 80·4 m.p.h. (*c*) 29·8 m.p.h.
(*d*) 26·3 m.p.h. (*e*) 49·8 m.p.h.
25. (*a*) 25·2 p. sec. (*b*) 49·5 p. sec. (*c*) 1·5 p. sec.
(*d*) 54·6 p. sec. (*e*) 62·9 p. sec.

EXERCISES 29. PAGE 83.

1. 24·3 sec. 2. 5·06 sec. 3. $2\frac{1}{2}$ sec. 4. 130
5. 62 min. 6. 2 hrs. 14 min. 7. 1 hr. 31 min.
8. (*a*) 0·8 (*b*) 24 (*c*) 74
(*d*) 215·6 (*e*) 1·8 (*f*) 24·3
9. 4 sec. 10. 51 m.p.h. 11. 25:17 12. Second
13. Yes. 14. $2\frac{3}{4}$ m.p.h. (app.)
15. A little over $32\frac{1}{2}$ m.p.h.

EXERCISES 30. PAGE 86.

1. $1\frac{7}{8}$ hrs.
2. $1\frac{1}{2}$ days
3. 2 hrs. 45 min.
4. $8\frac{4}{7}$ hrs.
5. 2 yrs.
6. $1\frac{2}{3}$ days
7. $6\frac{2}{13}$; $26\frac{2}{3}$; $11\frac{4}{7}$ days
8. $8\frac{8}{9}$ hrs.
9. 6 min. more
10. 3 min. more from the time both are opened
11. Rs 591·67
12. $7\frac{1}{2}$ wks; 5 : 3
13. 30 days
14. $5\frac{1}{2}$ days
15. $2\frac{7}{9}$ times as fast

EXERCISES 31. PAGE 90.

1. 42 mi. from C
2. 1 hr. 24 min.
56 miles from the starting point of the 1st car
3. 35 km/hr.
4. 60 km/hr.
5. 2 hrs. $19\frac{1}{2}$ min.
$127\frac{7}{8}$ mi.
6. 2 hrs. $30\frac{3}{4}$ min.
$138\frac{3}{16}$ mi.
7. 1 hr. $38\frac{1}{4}$ min.
$117\frac{9}{16}$ mi.
8. 50 km/hr.
9. $12\frac{6}{7}$ min.
10. In 2 hrs. 35 min.
$85\frac{1}{4}$ mi. from A
11. $2\frac{2}{11}$ hrs.
12. 2 km/hr.
13. 100; 1·8; $1\frac{1}{3}$
14. $20\frac{5}{6}$ m.p.h.
15. $1\frac{1}{80}$ sec.

EXERCISE 32. PAGE 91.

1. $8\frac{7}{16}$ sec.
2. 45 m.p.h.
3. 94 yds
4. $66\frac{2}{3}$ yds
5. 11 sec.
6. 24 sec. (v. nearly).

EXERCISES 32 (contd.)

7. 23 sec. (v. nearly)
8. 3·5 sec.
9. 3·7 sec.
10. $4\frac{3}{8}$ sec.
11. $3\frac{3}{5}$ sec.
12. 69 yds
13. 44; 88 yds
14. $6\frac{1}{4}$ sec. (v. nearly).
3·13 sec.
5·76 sec.
9 sec.
15. 84 yds.
30 m.p.h.

EXERCISES 33. PAGE 93.

1. $\frac{1}{2}, \frac{3}{5}, \frac{3}{4}, \frac{1}{8}, \frac{1}{3}, \frac{7}{20}, \frac{x}{100}$

2. 40%, 15%, $12\frac{1}{2}$%, $16\frac{2}{3}$%, 150%, $\frac{100x}{y}$ %

3. (*a*) $17\frac{1}{2}$% (*b*) $18\frac{3}{4}$% (*c*) 9%
(*d*) $1\frac{1}{4}$% (*e*) $31\frac{1}{4}$%
4. 95%, 5%
5. £16
6. £125
7. 4%
8. $14\frac{1}{2}$%
9. Rs. 6400
10. Rs. 4091
11. 300
12. 44%
13. 6:5
14. $9\frac{2}{3}$
15. 17·3%
16. 15%
17. 40% decrease.
18. $16\frac{2}{3}$%
19. $66\frac{2}{3}$; $83\frac{1}{3}$
20. 1%

EXERCISES 34. PAGE 98.

1. (*i*) For loss of 10%, C.P. $=\frac{100}{90}$ of S.P.

,, gain of 10%, C.P. $=\frac{100}{110}$ of S.P. etc.

(*ii*) For loss of 10%, S.P. $=\frac{90}{100}$ of C.P.

,, gain of 10%, S.P. $=\frac{110}{100}$ of C.P. etc.

(*iii*) For loss of 10%, loss $=\frac{10}{90}$ of S.P.

,, gain of 10%, profit $=\frac{10}{110}$ of S.P. etc.

(*iv*) For loss of 10%, C.P. $=\frac{100}{10}$ of loss.

,, gain of 10%, C.P. $=\frac{100}{10}$ of profit etc.

EXERCISES 34 (contd.)

3. (*a*) Rs 231 (*b*) Rs 120 (*c*) Rs 276
(*d*) Rs 204 (*e*) Rs 200 (*f*) 300 fr.
(*g*) £300; £231 (*h*) £532

EXERCISES 35. PAGE 98.

1. (*a*) Rs 28·04
Rs 378·54
(*b*) Rs 60·90
Rs 730·80
(*c*) Rs 1092
Rs 4108
(*d*) £360
£5693 6*s*. 8*d*.
2. (*a*) profit of 11%
(*b*) profit of 15%
(*c*) profit of $3\frac{1}{2}$%
(*d*) loss of $8\frac{1}{5}$%
3. (*a*) Rs 73·40
(*b*) Rs 832
(*c*) Rs 1360
(*d*) $758·76
4. Rs 1000
5. Rs 90
6. Rs 60
7. 38·01%
8. 324 fr.
9. 3502·8 fr.
10. 14%
11. 4·98% loss
12. 5·75% gain
13. Rs 62·40
14. $46·80
15. 13·04%

EXERCISES 36. PAGE 100.

1. 20%
2. Rs 2·40 a doz; 48·14%
3. 122·22%
4. $51\frac{3}{7}$%
5. Rs 6·25
6. $14\frac{4}{5}$%
7. $8\frac{1}{3}$%
8. £37 1*s*.
9. $12\frac{1}{2}$%
10. 44·7%
11. $1\frac{1}{3}$%
12. 80%

EXERCISES 37. PAGE 102.

1. 83 nP. in the Re
Rs 4166·67
Rs 3,333·33
2. Rs 5250
3. Rs 7888·89
4. $62\frac{1}{2}$ nP.
5. 13*s*. in the £
£209 2*s*. 9*d*.
6. £1042 2*s*. $4\frac{1}{2}$*d*.
7. $60\frac{5}{12}$%
8. 58·2% (approx.)

Exercises 38. Page 106.

1. Rs 85·75 2. Rs 47·26 3. Rs 13·14
4. Rs 225 5. £ 393 6. Rs 2005·60
7. Rs 755·30 8. Rs 402·50 9. £201
10. £11624·50

Exercises 39. Page 109.

1. (*a*) Rs 1296 (*b*) £57312. (*c*) £573 12*s*.
(*d*) Rs 183·75 (*e*) £6 15*s*.
2. (*a*) 6% (*b*) 2% (*c*) 7½%
(*d*) 4½% (*e*) 2¾%
3. (*a*) 2 yrs. (*b*) 3½ yrs. (*c*) 4 yrs.
(*d*) 2½ yrs. (*e*) 5¼ yrs.
4. (*a*) Rs 225 (*b*) Rs 620 (*c*) £7315
(*d*) Rs 2450 (*e*) £330 12*s*. 6*d*.
5. (*a*) 3% (*b*) 5% (*c*) 5 yrs
(*d*) 2 yrs (*e*) 2 yrs (*f*) 4%
6. (*a*) £150 (*b*) Rs 285 (*c*) £122 10*s*.
(*d*) £142 10*s*. (*e*) £620

Exercises 40. Page 110.

1. £60 at 4%; 6*d*. 2. Rs 70·85 3. 20 yrs. 20 yrs. 4. 4⅙% 5. 5⅓%
6. ½*d*. 7. £1496 5*s*. 6*d*. 8. £170 10. Rs 1100
11. Rs 8·33 12. Rs. 2752 13. Rs 23,350 14. Rs 11200
15. £9862 10*s*.

Exercises 41. Page 115.

1. 8·2296 2. 6·6297 3. 8·6692 4. 7·6629
5. 6·9312 6. 6·0697 7. 7·0230 8. 8·5138
9. 8·6787 10. 6·9106 11. 3·2611 12. 2·8632
13. 1·3664 14. 1·8453 15. 1·8181 16. 5·7363
17. 7·5849 18. 8·4116 19. 4·5774

Exercises 42. Page 118.

1. 6165 3. 318·4 5. 2467
2. 799·4 4. 269·5 6. 622

EXERCISES 42. (contd.)

7. 17790
8. 114·5
9. 6·572
10. 6·362
11. 8·023
12. 4·952
13. 540.1
14. 100.2
15. 5121
16. 513100
17. 2625
18. 4·956
19. 13·78
20. 6·524
21. 3·277
22. 42·75
23. 9·554
24. 4·429
25. 24.88.

EXERCISES 43. PAGE 122.

1. 345·4
2. 0·3454
3. 22·22
4. 0·01397
5. 0·0002331
6. 0·00006407
7. 15·75
8. 0·008555
9. 1·359
10. 1650
11. 1·414
12. 0·7563
13. 0·1369
14. 0·007656
15. 0·001879
16. 0·01916
17. 0·05585
18. 0·102
19. 0·08945
20. 0·2683
21. 0·034
22. 0·1393
23. 0·1932
24. 0·2646
25. 0·1061
26. 0·3473
27. 0·9232
28. 4·672
29. 37·37
30. 4·481
31. 30·26
32. 1041000
33. 0·029
34. 15·4
35. 0·06091
36. 0·000195
37. 731·1
38. 3·202
39. 11·43

EXERCISES 44. PAGE 123.

1. Read $T=2\pi\sqrt{(l\div g)}$; 1·005
2. 7·55
3. 127.7
4. 310
5. 7261 m
6. $\mu=3{\cdot}07$
7. 89·798
8. 1305
9. (*a*) 2; (*b*) 4; (*c*) $\frac{1}{2}$; (*d*) 2; (*e*) 1; (*f*) 3
10. 574·1.

EXERCISES 45. PAGE 125.

1. (*a*) Rs 161·10 (*b*) Rs 1756.85 (*c*) Rs 177·90
(*d*) $172 38 cents (*e*) 7611 fr. 78 cents
2. Rs 437·60
3. £135 6*s*. 10*d*.
4. $214 67 cent.
5. Rs 285

EXERCISES 45. (contd.)

6. £68 15*s*. 7. 20025 fr.
8. Rs 568·75 9. 2400 at 3½%
7600 at 4½%

EXERCISES 46. PAGE 130.

1. (*a*) Rs 743.75 (*b*) Rs 977·94
(*c*) Rs 936·50 (*d*) Rs 1087·70
(*e*) Rs 2032·13 (*f*) Rs 1731·06
2. (*a*) £988 9*s*. 10*d*. (*b*) £1,040 17*s*. 1*d*.
(*c*) £9,663 6*s*. (*d*) £8,898 16*s*. 2*d*.
3. 5% 4. 6% 5. 4½% 6. 7%

EXERCISES 47. PAGE 134.

1. Rs 182·70
2. Rs 306·04
3. Rs 658·36
4. Rs 700·67
5. Rs 811·62
6. Rs 1891·50
7. Rs 3150·00
8. Rs 1854·54
9. Rs 306·04
10. Rs 649·09
11. Rs 487·20
12. Rs 494·59
13. Rs 624·32
14. Rs 627·54.

EXERCISES 48. PAGE 136.

1. (*a*) Rs 786·64 (*b*) Rs 815·38 (*c*) Rs. 672·38
(*d*) Rs 886·13 (*e*) £617 11*s*. (*f*) £1,403 8*s*. 8*d*.
(*g*) 2012 fr. (*h*) 7764 fr. 73 cent.
(*i*) $1036 37 cent (*j*) $701 42 cent.
2. (*a*) £15 5*s*. 1*d*. (*d*) Rs 13·60
(*b*) Rs 27·65 (*e*) 238 fr. 70 cents
(*c*) Rs 176·44 (*f*) $539 17 cents.

	Half Yearly	Yearly	Difference
3. (*a*)	Rs 846·94	Rs 843·26	Rs 3·68
(*b*)	Rs 644·33	Rs 639·45	Rs 4·88
(*c*)	£388 9*s*. 6*d*.	£386 18*s*. 1*d*.	£1 11*s*. 5*d*.

EXERCISES 49. PAGE 141.

1. $12\frac{1}{2}$% 2. $29\frac{3}{8}$% 3. 7% 4. £92 11*s*. 10*d*.
5. $6\frac{2}{3}$% 6. $27\frac{1}{7}$% 7. 18:25 8. 6:5
9. £2 10. 34% 11. 35000; 52500
12. $30\frac{5}{8}$% 13. $9\frac{3}{4}$% 14. $9\frac{1}{4}$% 15. 15%
16. 30,000 17. 21 18. 3·04% 19. 16·64%
20. 29·96% 21. 9·2% 22. Decrease 0·25%
23. 16·64% 24. 27·12% 25. Decrease 1·44%
26. 52·1% 27. 25% 28. 20%
29. Decrease $16\frac{2}{3}$% 30. 112·4% 31. 25·55%
32. Rs 50·62 (app.)

EXERCISES 50. PAGE 148.

1. (*a*) 1 : 1 (*b*) 1 : 2 (*c*) 9 : 1 (*d*) 3 : 1
(*e*) 10:1:2 is one answer 2. 2 : 5
3. 7 : 11 4. 10 kg 5. Rs 2·75
6. $16\frac{2}{3}$ doz. 7. 45 doz. 8. 10 doz.
9. 6 of mixture to 1 of water
10. (*a*) 3 : 1; $31\frac{1}{4}$% (*b*) 4 : 5; 47% (*c*) 7 : 10
11. (*a*) 5 : 3; 23 : 5 (*b*) 2 : 1; $11\frac{2}{3}$%
(*c*) 7 : 13; 18% (*d*) 1 : 3; 7 : 9; 35%; 45%; 20%
12. (*a*) 2 : 4 : 9 (*b*) 1 : 5 : 9

EXERCISES 51. PAGE 151.

1. (*a*) 4540 gm. (*b*) 1590 gm. (*c*) 91·9 gm.
2. (*a*) 5240 gm. (*b*) 14400 gm. (*c*) 1010 gm. (*d*) 3260 gm.
(*e*) 819 gm.
3. (*a*) 17·2 lb. (*b*) 54·5 lb. (*c*) 85·2 lb. (*d*) 24·7 lb.
(*e*) 1·46 lb. (*f*) 0·83 lb.
4. (*a*) 1146 gm. (*b*) 2616 gm. (*c*) 308 gm. (*d*) 2420 gm.
5. (*a*) 16·75 lb. (*b*) 1·48 lb. (*c*) 99·5 lb. (*d*) 73·3 lb.
6. 144 lb. 7. 2·45 cm. 8. 583 gm.
9. 28·1 cm. 10. 1·85 in. 11. 1104·86 gm.
12. 0·1 cm. 13. 57·5 lb. 14. 7849·92 gm.
15. 6·2 lb. 16. 2611·3 gm. 17. 290

EXERCISES 51. (contd.)

18. 473
19. 2660
20. 1·16
21. 0·04 in.
22. 90
23. 72 km.
24. 20 in.
25. 0·68 ft.
26. 15·6 in.
27. 4·08 in.
28. 44·73 cm.
29. 14·05 cm.
30. 22·1 cm.

EXERCISES 52. PAGE 156.

1. 8·73 ft
2. 0·8″
3. $56\frac{12}{81}$
4. $4\frac{9}{20}$
5. $\frac{789}{160}$
6. $51\frac{4}{5}$; $50\frac{7}{15}$
7. $2\frac{1}{3}$; $2\frac{3}{10}$; the 1st
8. 1*s*. 2*d*.
9. 48 km p.h.; 48 km p.h.
10. $9\frac{27}{77}$ m.p.h.
11. $46\frac{2}{3}$ m.p.h.
12. 24 km.p.h.
13. 24 km p.h.
14. $\frac{3}{8}$
15. 2
16. 45
17. $50\frac{1}{4}$; $47\frac{4}{5}$
18. 34; 35
19. 12, 13, 14, 15, 16
20. 23-28
21. 54 in.

EXERCISES 53. PAGE 162.

1. 250 ft.
2. 14·28 m
3. 2·26″; 11·1″
4. 1·56 m
5. $\sqrt{3}$; $\frac{1}{\sqrt{13}}$.
6. (1) $\frac{1}{2}=\cdot 5$ (2) $\frac{\sqrt{3}}{2}=\cdot 86603$ (3) $\frac{\sqrt{3}}{2}=\cdot 86603$ (4) $\frac{1}{2}=0\cdot 5$
7. (i) 3·42; (ii) 4·756. (iii) 28°21 (iv) 24.63 (vi) 8°25′ (v) 72°33

Deg. Min.	*Sin*	Cos
8. 62°	·88295	·46947
35°	·57358	·81915
48°	·74314	·66913
57°	·83867	·54464
76°	·97030	·24192
85°	·99619	·08716
16°	·27564	·96126
27°	·45399	·89101

EXERCISES 53 (contd.)

Deg. Min.	*Sin*	*Cos*
45′	·01309	·99991
81°17′	·98845	·15155
72°14′	·95231	·28847
55°55′	·82822	·56040
36°37′	·59646	·80264
48°49′	·75261	·65847
31°39′	·52471	·85127
25°59′	·43811	·89892

9. tan A = ·75
cos A = ·80
10. cos A = $\frac{15}{17}$ = ·88
tan A = $\frac{8}{15}$ = ·54
11. 1 12. 1 13. $\frac{2m^2;}{m^2+n^2}$ $\frac{2m^2}{m^2-n^2}$
14. 0·261 cm. 15. 38°56′ 16. 78·3; 38·6, 41·1.
17. 57·8 m; 177·8 m 18. 8·45 19. 10, 8·7, 43·5 sq.cm.
20. 59·45 sq.cm.

EXERCISES 54. PAGE 168.

(i) $\frac{1}{\sqrt{3}}$ (ii) $\frac{1}{\sqrt{3}}$ (iii) $\frac{1}{2}$ (iv) 2; (v) $\frac{\sqrt{3}}{4}$ (vi) $\frac{\sqrt{3}}{4}$
(vii) 4 (viii) 1 (ix) 1 (x) 2

EXERCISES 55. PAGE 170.

1. (1) 28°39′; (ii) 41°11′ (iii) 46°45′ (iv) 33°14′
2. 48°1′ 3. 78°48′ 4. 27·92m; 11°10′
5. 14·26′, 10°19, 14°26′ 6. 46°4′, 56°19′
7. 63°47′; 38°24′ 8. 30°81′ 9. 4°59′.
10. 69°44′ 11. 39.72m 12. 70·7cm; 25°7′ 30°58′; 50cm.
13. 9°36′; 5°44′ 14. 58°; 48°31′; 5.34 in.
15. 36 sq.cm.; 13·9 cm; 67°23′.

EXERCISES 56. PAGE 178.

1. 3·61
2. 80°, 6·53 cm
3. 7 in.
4. 75°30′, 13·5″, 14·1″
5. 14·3 cm
6. 4·35′, A = 78°18′; B = 49°36′
7. A = 41°25′; B = 55°46′; C = 82°49′
8. The distance between the tips is 12·17″; 8.67″
9. $\frac{3}{5}$; $\frac{4}{5}$; $\frac{56}{65}$
10. 65°8′
11. 88°, 32′
12. 42°
13. 5·91 in; 20°30′
14. 31·4 cm
15. 38°22′

EXERCISES 57. PAGE 179.

1. 14·1 km
2. 2078 m
3. 53°45′; 392·3 m.
4. 15·32 km. ; 325°42′
5. 122°50′; 18·59 ft.
6. 19 Knots per hr. (appr.)
7. 3·88 cm
8. 24 km

TEST PAPERS

TEST 1.

1. $91\frac{3}{7}$; 128
2. $19\frac{11}{21}$; £1152 16*s.* 6*d.*
3. 8·36 cm; 555·9 sq.cm
4. 62·5%
5. 4%
6. 4·6 km; 213°31′.
7. 52°53′
8. 1·31 in; 0·86 in.

TEST 2.

1. $\frac{67}{107}$ 2. 40% 3. 2·35 4. Rs. 505·07
5. 5·8875 tons. 6. 36·7%; 80%; 80%; 58·3%
7. Rs 2244. 8. 3 hrs 48 min. 9. 100°17′ 10. 102·3 gm.

TEST 3.

1. 958000; 859500; 98·12.
2. $\frac{7}{17}$, $\frac{85}{17}$, $4\frac{7}{17}$
3. 0·0116; 0·426
4. 3·54 km; 25°8′

5. Rs. 1520; Rs 1680; $\frac{1}{4}$% loss
6. 1·125 cm
7. 39·24 cm
8. Rs 350; Rs 230; Rs 470
9. Rs 15549
10. 83·14 cm; 36°59

TEST 4.

1. (*a*) $\frac{8}{81}$; (*b*) £867 1*s*. 11*d*.
2. £2 8*s*. 4*d*.
3. 18 days.
4. 8.
5. £350.
6. Rs 63·25
7. 27°53′; 7·5 cm.
8. 86 m; 49 m.
10. 34$\frac{2}{7}$ min.

TEST 5.

1. Rs 653046·3.
2. 74$\frac{1}{2}$%
3. 3·2 cm
4. 97·53%
5. 84 gal.
6. £3550.
7. 57·125 cc; 34·28 gm.
8. 71°34′; 12·3 ft.
9. 4·65; 0·913.
10. 13 ft; 38°13′

TEST 6.

1. 9·11 oz. £2 7*s*. 10*d*.
2. (i) 0·0269.
   (ii) 0·477.
3. 38°22′.
4. 2464 sq.m.
5. 3:4.
6. 12 sec, 70 m.
7. 14·9 cm.
8. Rs 25·96
9. Rs 120; Rs 84·50.
10. $\pm\frac{60}{61}$; $\pm\frac{11}{60}$

TEST 7.

1. (*a*) $\frac{3}{2}$ (*b*) £70 4*s*. 7·5*d*.; (*c*) 2640 yds.
2. 1728; $\frac{1}{216}$
3. 4·379. 4. 48 km 5. £57 7*s*. 6*d*.
6. Rs 800, Rs 950, Rs 1250. 7. 119$\frac{3}{48}$, 39$\frac{3}{48}$. 8. Rs 15.
9. 48′
10. 24°54′; 24°54′; 130°12′.

TEST 8.

1. (*a*) 490000; (*b*) 1168800; (*c*) 264·44
2. 0·3581; 0·740
3. 63·7%
4. 7·86 km
5. 360 sq.in., 13·9 in., 67°23′
6. 8¼, 5940.
7. 7·43 cm., 12·8 cm., 11·8 cm
8. 1·53 lb.
9. 1·24 cm; 11·8 sq.cm
10. 18·19 sq.cm.

TEST 9.

1. Rs 58·70 2. 31·25, 19. 3. 8·1 4. 11·5 kg
5. 6 cc. (*a*) (i) 38·627 (ii) ·39600 (*b*) 1·1821. 7. Rs 1954·6
8. 6:1 9. 14°40′ 10. 238 sq.in.

TEST 10.

1. 16 days, £1.
2. 3·31 cm.
3. Rs 360.
4. 8:10:27.
5. 57·2.
6. 5376; Rs 134·40.
7. 2632 cc.
8. 168·8°F; 40°C
9. 4·25; 0·806
10. 21·4 sq.cm.